Yield Modeling

Here yield model refers to the effect of randomly distributed defects on integrated circuit chips. The problem of determining the yield of good chips ("good" means that they pass all parametric and functional tests that are specified for the product) is identical to the classical statistics problem of placing n balls in N cells, and then calculating the probability that a given cell contains k balls. Let us denote the probability that a given chip contains k defects as P_k, then P_k can be expressed as

$$P_k = \int_0^\infty e^{-m} \frac{m^k}{k!} f(D) dD, \tag{1}$$

where $m = n/N = DA$, A is the total area of the chip, and $f(D)$ is the normalized distribution function of defect density D, $\int_0^\infty f(D)dD = 1$. Now the modeling of yield is to find the appropriate distribution function. In general the yield of a chip is P_0. Of course we consider killer defects only. The most common distribution functions used in yield modeling are the following:

- Delta function:

$$f(D) = \delta(D_0), \quad P_0 = e^{-D_0 A}$$

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- Triangular function:

$$f(D) = \begin{cases} D/D_0^2 & 0 < D < D_0, \\ 2/D_0 - D/D_0^2 & D_0 < D < 2D_0, \\ 0 & \text{otherwise.} \end{cases}, \quad P_0 = \left\{ \frac{1 - e^{-D_0 A}}{2D_0 A} \right\}^2$$

- Rectangular function:

$$f(D) = \begin{cases} 1/2D_0 & 0 < D < 2D_0, \\ 0 & \text{otherwise.} \end{cases}, \quad P_0 = \frac{1 - e^{-2D_0 A}}{2D_0 A}$$

- Gamma function:

$$f(D) = \frac{1}{\Gamma(\alpha)\beta^\alpha} D^{\alpha-1} e^{-D/\beta}, \quad P_0 = \frac{1}{(A\beta + 1)^\alpha},$$

  where the average density of defects is given by $D_0 = \alpha\beta$ and the variance of D is given by $\alpha\beta^2$.

For MOS memory chips designed with redundant circuitry which can be used to replace defective circuit elements, the chip will still be good if it has some defects which are repairable. For a memory chip with L redundant columns, the yield will be

$$Y = P_0 + \sum_{i=1}^{L} \eta_i P_i,$$

where η_i is the probability that a chip containing i defects can be repaired by using the L redundant columns.