chapter 1 Space, Time, and Mass

Coordinates and reference frames (Section 1.1)

1. From x'–y' coordinates to x–y coordinates
   - Context in the textbook: This is an exercise related to Fig. 1.3 of Section 1.1.

The unit of length (Section 1.2)

2. Modern standard of length: 1 m
   - Context in the textbook: After Eq. (1.1) of Section 1.2.
3. Light-year

The unit of time (Section 1.3)

4. Working of a cesium clock
   - Context in the textbook: This is an exercise after the discussion of the cesium clock in Section 1.3.

The unit of mass (Section 1.4)

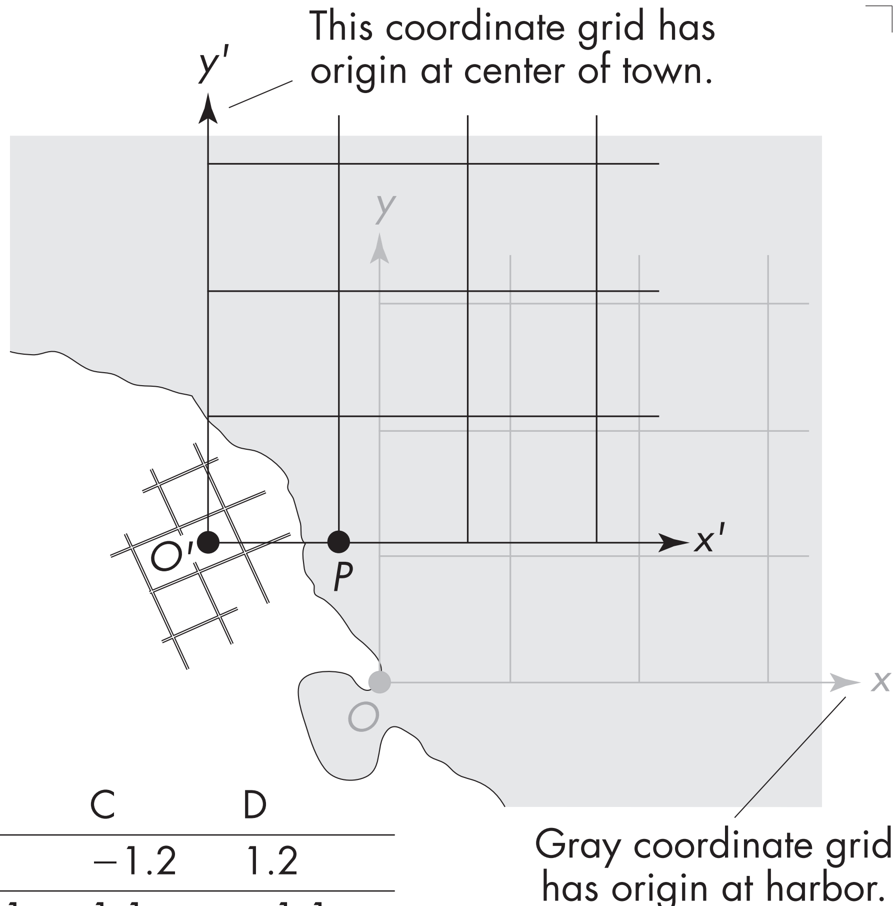
5. Compare two metal blocks
   - Context in the textbook: Before Example 2 in Section 1.4.

Consistency of units (Section 1.6)

The topic of dimensional analysis is mentioned only in passing after Eq. (1.8) of Section 1.6. The following two questions are to supplement the discussion of this topic.

6. Dimensional analysis: Twirling a ball
7. Dimensional analysis: A rope

Consider the two sets of coordinate grids shown. The first set is the x–y coordinate system with red grids. The origin is at O. The shifted set is the x'–y' system with the origin O'. From the figure, determine the x–y coordinates of the point O':



	A	B	C	D
$x_{O'}$	−1.2	1.2	−1.2	1.2
$y_{O'}$	1.1	−1.1	1.1	−1.1

Extra: Given the point P at $(x',y') = (1,0)$, determine the x–y coordinates of P: x_P and y_P.

Explanation: Notice that O' is to the left of O, so $x_{O'} < 0$. O' is above O, so $y_{O'} > 0$. Answer = A.

Explanation—extra: First verify the general relations for relating x'–y' coordinates to x–y coordinates for any particular point: $x = x' + x_{O'}$ and $y = y' + y_{O'}$. Because the x'–y' coordinates of point are $(x',y') = (1,0)$ and the x–y coordinates of O' are $(x_{O'},y_{O'}) = (-1.2,1.1)$, at P these relations lead to $x_p = 1 - 1.2 = -0.2$ and $y_p = 0 + 1.1 = 1.1$.

The speed of light in free space is $c = 3.0 \times 10^8$ m/sec.
Find the time Δt for light to travel 1 meter.
Choose one (in units of seconds):

	A	B	C
Δt	3.3×10^{-9}	3.3×10^{-8}	3.3×10^{-7}

Hint: speed = distance/time.

Explanation: speed = distance/time. From this we get Δt = distance/speed = 1 m/(3×10^8 m/s) = 3.3×10^{-9} s. Answer = A.

One light-year (ly), a common length unit in astronomy, is defined to be the distance that light in a vacuum travels in one year. Using the speed of light $c = 3 \times 10^8$ m/s and one year = $365 \times 24 \times 60 \times 60 \approx 3 \times 10^7$ s, the order of magnitude of one light-year is which of the following?

	A	B	C	D
1 light-year in m	$\sim 10^7$	$\sim 10^8$	$\sim 10^{15}$	$\sim 10^{16}$

Explanation: 1 ly = $ct = 3 \times 10^8 \times 3 \times 10^7 \sim 10^{16}$. Answer = D.

3. Light-Year

A cesium clock achieves its high level of accuracy by keeping track of internal vibrations. In this process every second of time corresponds to approximately 9.2×10^9 vibrations. Suppose that such a clock is so accurate that it gains only 1 second every 20 million years. For this clock, determine the corresponding amount of change in the number of vibrations per second:

A	Gain of 10^{-5} vibrations in 1 second
B	Loss of 10^{-5} vibrations in 1 second
C	Gain of 10^5 vibrations in 1 second
D	Loss of 10^5 vibrations in 1 second

Hint: 20×10^6 yrs $\sim 20 \times 10^6 \times (3 \times 10^7) = 6 \times 10^{14}$s.

Explanation: Denote the priod of the ideal clock by To, and of the actual clock To′. The actual clock ticks faster, or To′ < To. In $t = 2 \times 10^7$ yrs, the ideal clock undergoes t/To vibrations and theoretical clock t/To′ vibrations. Their difference worth the vibrations in 1 sec, i.e. No $= 9.2 \times 10^9$ vibrations. Put all together t/To′ − t/To = No. In one second the gain in vibration is given by $1/To' - 1/To = No/t = 9.2 \times 10^9/6 \times 10^{14} \sim 10^{-5}$ vibrations. Answer = A.

Consider two solid blocks that are identical in size and shape:

- One is made of copper. It has a density $\rho_{Cu} = 8.9\ kg/m^3$ and an atomic mass $m = 59u$.
- The other is aluminum. It has a density $\rho_{Al} = 2.7\ kg/m^3$ and an atomic mass $m = 13u$.

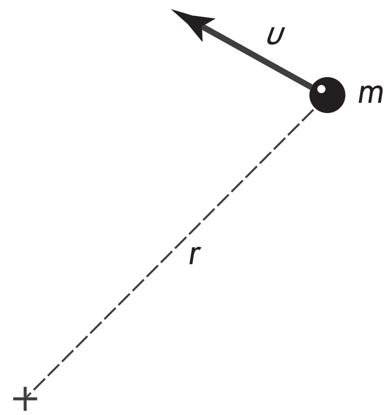
Compare their masses M, and the total number of atoms N, in each block:

	A	B	C	D
M_{Cu} vs. M_{Al}	$>$	$>$	$<$	$<$
N_{Cu} vs. N_{Al}	$>$	$<$	$>$	$<$

Explanation:

- Compare their masses: Because the volumes of the two blocks are the same, density $\rho = M/V$ implies that $M_{Cu}/M_{Al} = \rho_{Cu}/\rho_{Al} = 8.9/2.7 > 1$.
- Compare their total number of atoms: $N = M/m$. This leads to $N_{Cu}/N_{Al} = (M_{Cu}/m_{Cu})/(M_{Al}/m_{Al}) = (M_{Cu}/M_{Al})/(m_{Cu}/m_{Al}) = (\rho_{Cu}/\rho_{Al})/(m_{Cu}/m_{Al}) = (8.9/2.7)/(59/13) = 3.3/4.5 < 1$. Answer = B.

5. Compare Two Metal Blocks



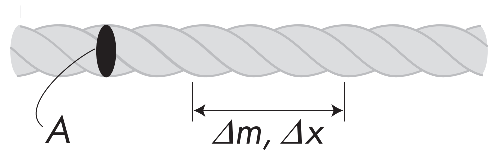
While boy is twirling a ball he is pulling the string at the same time. The force pulled by the boy provides centripetal force. (Later you will learn that force equals mass times acceleration.) Intuitively we expect that the centripetal force should depend on r, m, and v. Assume it has the form $F = k\, m^x r^y v^z$, where k is dimensionless. Determine the values for the exponents x, y, and z.

Choose one of the following:

A	$x = 1,\ y - z = 1,\ z = -2$
B	$x = 1,\ y + z = 1,\ z = 2$
C	$x = 1,\ y + z = 1,\ z = -2$

Hint: [force] = [mass] $\times$ [acceleration] = $[M][L]/[T]^2$, $[km^x r^y v^z] = [M^x L^y (L/T)^z]$, where M, L, and T represent respectively the mass, length, and time quantities used in dimensional analysis.

Explanation: $[M][L]/[T]^2 = [M]^x[L]^y\,[L]^z/[T]^z$. By equating the powers of $[M]$, $[L]$, and $[T]$, $x = 1$, $y + z = 1$, and $z = 2$. Answer = B. Notice that the solution of the simultaneous equations gives $x = 1$, $y = -1$, $2 = 2$, or $F = kmv^z/r$.



Consider a rope with a uniform cross section. If the rope's density $\rho = 1$ kg/m^3 and its cross-sectional area $A = 0.1\text{m}^2$, find its linear mass density: μ = mass/length = $\Delta m/\Delta x$ in terms of ρ and A.
Choose one of the following:

	A	B	C
μ	ρA	ρ/A	A/ρ

Hint: Write $\mu = \rho^x A^y$. Determine the powers x and y based on dimensional analysis.

Explanation: $[\mu] = M/L = [\rho^x A^y] = (M/L^3)^x(L^2)^y$. Equating powers of M and L leads to $1 = x$ and $-1 = -3x + 2y$.

Substituting $x = 1$ into the second equation gives $-1 = -3 + 2y$. So $y = 1$, or $\mu = \rho A$. Answer = A.

 7. Dimensional Analysis: A Rope