chapter 2 Motion along a Straight Line

Average speed and average velocity (Section 2.2)

1. Velocity versus speed
   - Context in the textbook: After Example 2.

Instantaneous velocity and instantaneous acceleration (Sections 2.3, 2.4)

2. Acceleration curve

Motion with constant acceleration (Section 2.5)

3. Displacement curves
4. Collision time without deceleration

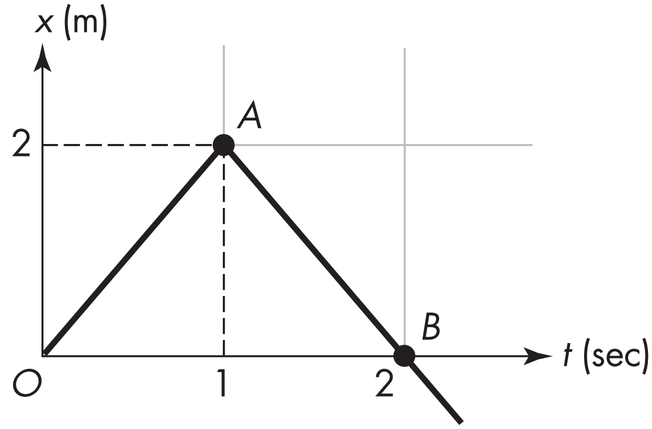
The acceleration of free fall (Section 2.6)

5. Acceleration at the top
6. Maximum height
7. Rising ball
8. Timing a rising ball by height
9. A ball passes by a window
   - Supplementary problems for this section.

Integration of equations of motion (Section 2.7)

10. Average velocity
    - This is an exercise on the notion that "distance traveled is the area under velocity curve" as implied by Eq. (2.35).

The following figure shows the displacement versus time graph of a particle moving along the x axis:



Determine the average velocity and the average speed (in units of m/s) of the particle along the whole path shown from O to A to B.

	A	B	C	D
Average velocity	2	0	2	0
Average speed	2	2	0	0

Extra: Determine the instantaneous velocity and the instantaneous speed at B.

Explanation:

The average velocity is given by

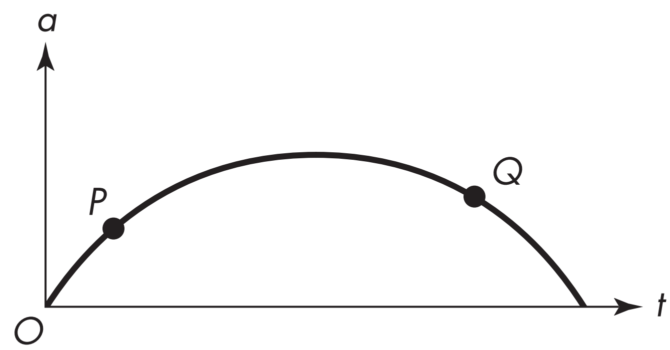
- $\bar{v} = \dfrac{\text{displacement}}{\text{time}} = \dfrac{x_B - x_O}{t_B - t_O} = 0.$

The average speed is given by

- $\text{average speed} = \dfrac{\text{distance}}{\text{time}} = \dfrac{|x_A - x_O| + |x_B - x_A|}{t_B - t_O} = 2 \text{ m/s}.$

Answer = B.

Explanation—extra: At B the instantaneous velocity is given by the slope of the displacement curve. From the plot we see that the slope is given by $v = (x_B - x_A)/(t_B - t_A) = (0 - 2)/(2 - 1) = -2$ m/s. The instantaneous speed is the magnitude of the velocity, which is 2 m/s.



The diagram describes the acceleration versus time behavior for a car moving in the *x* direction. At the point *P* the car is moving how?

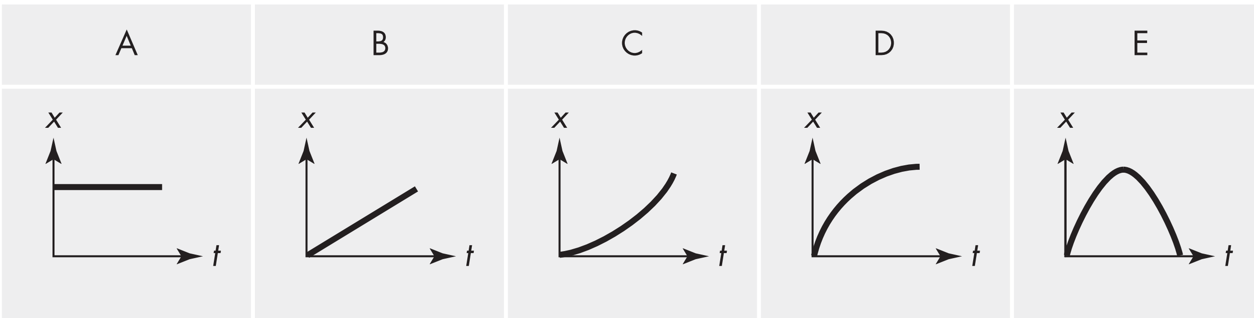
A	With an increasing velocity
B	With a constant velocity
C	With a decreasing velocity

Extra: Describe the change of velocity at *Q*.

Explanation: $a = dv/dt$. As long as the acceleration is positive, the velocity is always increasing. Answer = A.

Explanation—extra: Here again $a > 0$, so the velocity continues to increase.

A car starting from the origin is moving along the positive x direction with a constant velocity. Which graph correctly describes the x coordinate versus time for this motion?



Extra: Which graph represents the x coordinate of a constant acceleration starting from $x = 0$?

Explanation: The motion along the x axis with a constant velocity v is described by $x = vt$. Answer = B.

Explanation—extra: A constant acceleration starting from $x = 0$ is described by $x = \frac{1}{2}at^2$. Answer = C.

3. Displacement Curves

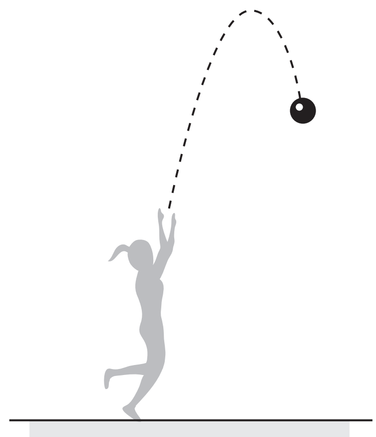
Refer to Example 8 of Section 2.5 in the textbook. The initial velocity of the minivan is $v_0 = 22.2$ m/s; the initial velocity of the truck is $v'_0 = 6.9$ m/s; and the initial distance between the two vehicles is $x'_0 = 12$ m. Find an expression for the collision time if the minivan does not decelerate. Choose one of the following:

A	B	C	D
$\dfrac{x'_0}{v_0}$	$\dfrac{x'_0}{v'_0}$	$\dfrac{x'_0}{v_0 - v'_0}$	$\dfrac{x'_0}{v_0 + v'_0}$

Extra: Verify that the collision time obtained here (without any deceleration) is shorter than the collision time when the brake is applied in Example 8, which is $t_c = 1.1$ s.

Explanation: In Example 8, if we set $a = 0$, the collision condition leads to the equation $v_0 t = x'_0 + v'_0 t$. Solving for t gives $t = x'_0/(v_0 - v'_0)$. Answer $= C$.

Explanation—extra: Using the numbers given, $t = x'_0/(v_0 - v'_0) = 12/(22 - 6.9) \approx 0.8 \text{ s} < t_c = 1.1$ s, which shows that when the brake is applied, the collision takes place at a later time.



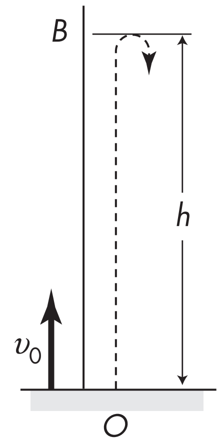
When a stone is thrown straight upward, its velocity at its highest point is zero. What is its acceleration at this point?

	A	B	C
Acceleration at top	9.8 m/s^2 down	0	9.8 m/s^2 up

Explanation: Near the surface of the Earth, for all practical purposes the gravitational acceleration is a constant 9.8 m/s^2 pointing downward. Answer = A.

To illustrate how this works, let us take for example an upward initial velocity of 9.8 m/s. After the first second the velocity will be 0. After the second second the velocity will be 9.8 m/s, but heading down. This can be written as −9.8 m/s, where the minus sign indicates the downward velocity. In other words, in each second the velocity is decreased by 9.8 m/s.

5. Acceleration at the Top



Find the maximum height h (in m) of a ball thrown vertically upward with an initial velocity $v_0 = 30$ m/s (assume $g \sim 10$ m/s^2:

	A	B	C
h	45	60	90

Hint: With the gravitational acceleration $a = -g$, its velocity and its displacement at locations O and B are

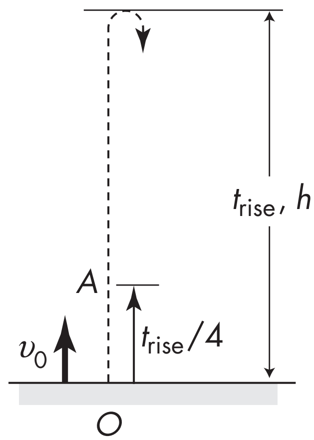
Location	Velocity	Displacement
O	v_0	O
B	0	h

Rewrite Eq. (2.25) of Section 2.5 as $v^2 = v_0{}^2 + 2ay$.

Extra: Compare the acceleration of the ball at the point O (just after being released from the thrower) to that at the point B.

Explanation: At B, $y = h$. At this point $v^2 = v_0{}^2 + 2ah$ gives $0 = v_0{}^2 + 2(-g)h$. So $h = v_0{}^2/(2g) = 30^2/(2 \times 10) = 45$ m. Answer = A.

Explanation—extra: The acceleration throughout the motion is $a = -g$, including both the initial point O and B at the top.



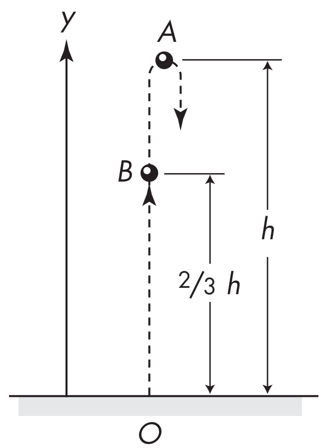
Consider a rising ball problem. The initial velocity of the ball is v_0. The time it takes for it to rise to its maximum height h is $t_{rise} = v_0/g$. Determine the velocity v_A at some intermediate point A, where $t_{OA} = t_{rise}/4$:

	A	B	C	D
v_A	$v_0/4$	$v_0/2$	$3v_0/4$	v_0

Extra: How high is the point A? Express OA in terms of h.

Explanation: At A, $v_A = v_0 - g(t_{rise}/4)$, where $t_{rise} = v_0/g$. This implies that $v_A = v_0 - gv_0/(4g) = 3v_0/4$. Answer = C.

Explanation—extra: To find OA we use the relation $v^2 = v_0{}^2 + 2ay$ (see Eq. 2.25 in the text for the present case, the one dimensional coordinate variable x is replaced by y, and the relation x_0 is used.). At A this relation gives $(3v_0/4)^2 = v_0{}^2 - 2g(OA)$. Solving for OA gives $OA = (1 - 9/16)v_0{}^2/(2g) = 7h/16$. In the last step, $h = v_0{}^2/(2g)$ was used.



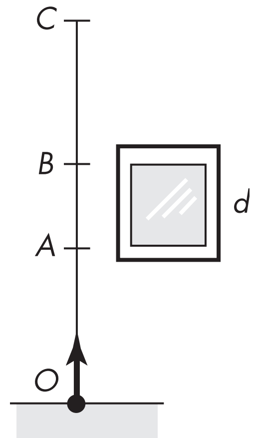
A ball is thrown vertically upward at $t = 0$ from the ground at point O. It reaches a maximum height h at point A. The height at point B is $2h/3$. Find the ratio of t_{BA} to t_{OA} (that is, the ratio of the time to go from point B to A to the time to go from point O to A):

	A	B	C	D
t_{BA}/t_{OA}	1/3	$1/\sqrt{3}$	1/2	$\sqrt{(2/3)}$

Extra: Show that $v_B = \sqrt{(2/3)gh}$.

Explanation: Consider the fall of the ball starting at rest at A. Eq. (2.22) implies that $AB = \frac{1}{2}g{t_{AB}}^2$, or $t_{AB} = \sqrt{2 \cdot AB/g}$, and $t_{AO} = \sqrt{2 \cdot AO/g}$. Thus $t_{AB}/t_{AO} = \sqrt{AB/AD} = \sqrt{(k/3)/h} = 1/\sqrt{3}$. Eq (2.25) together with Eq (2.23) implies that at any height the upward speed is the same as the downward speed. In other words, at any point the time taken for the ball to reach the top is the same as the time taken for the ball to fall from the top to that point. In other words, $t_{BA}/t_{OA} = t_{AB}/t_{AO} = 1/\sqrt{3}$.
Answer = B.

Explanation—extra: ${v_B}^2 = 2g(AB) = 2g(h/3)$, or $v_B = \sqrt{(2/3)gh}$.



Consider a ball thrown upward from the ground at point O. It passes by a window with height $AB = d$ in the time interval $t_{AB} = T$. The choice of the correct pair among the following four equations will let us solve for the two unknowns v_A and v_B. The equations are

(1) $v_A + v_B = 2d/T$ (2) $v_A - v_B = gT$

(3) $v_A + v_B = d/T$ (4) $v_B - v_A = gT$

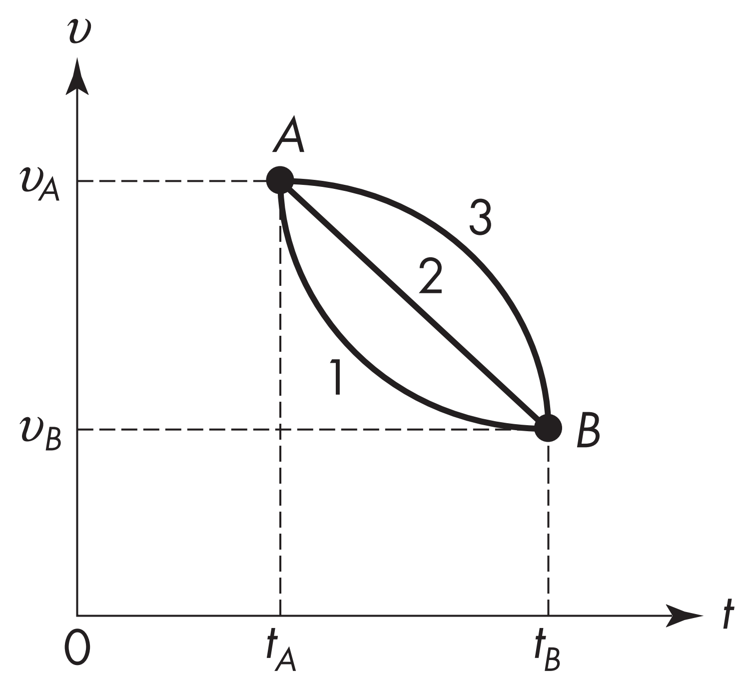
	A	B	C	D
Correct pair	(1) and (2)	(2) and (3)	(3) and (4)	(1) and (4)

Extra: Given $d = 2$ m and $T = 1$ s, find v_A. Assume $g = 10$ m/s^2.

Explanation: The average velocity is given by $\frac{1}{2}(v_A + v_B) = d/T$. So (1) is correct and (3) is incorrect. Because $v_A > v_B$, (2) is correct and (4) is incorrect. Answer = A.

Explanation—extra: [Eq. (1) + Eq. (2)]/2 = $v_A = 0.5(2d/T + gT) = 0.5(2 \times 2/1 + 10 \times 0.5) = 4.5$ m/s.

Consider the three velocity curves between points A and B shown here: Choose the correct relationship among quantities $\bar{v}_1$, $\bar{v}_2$, $\bar{v}_3$, and $(v_A + v_B)/2$, where $\bar{v}_1$, $\bar{v}_2$, and $\bar{v}_3$ represent the average velocities over the respective paths:



A	$\bar{v}_1 = \bar{v}_2 = (v_A + v_B)/2 = \bar{v}_3$
B	$\bar{v}_1 < \bar{v}_2 = (v_A + v_B)/2 < \bar{v}_3$
C	$\bar{v}_1 > \bar{v}_2 = (v_A + v_B)/2 > \bar{v}_3$

Explanation:

$$\bar{v} = \frac{\text{displacement}}{\text{time}} = \frac{\text{area under } v \text{ versus } t \text{ curve}}{t_B - t_A}$$

By inspection $\text{area}_1 < \text{area}_2 < \text{area}_3$; in turn $\bar{v}_1 < \bar{v}_2 < \bar{v}_3$. Answer = B.