chapter 4 Motion in Two and Three Dimensions

Projectile motion (Section 4.3)

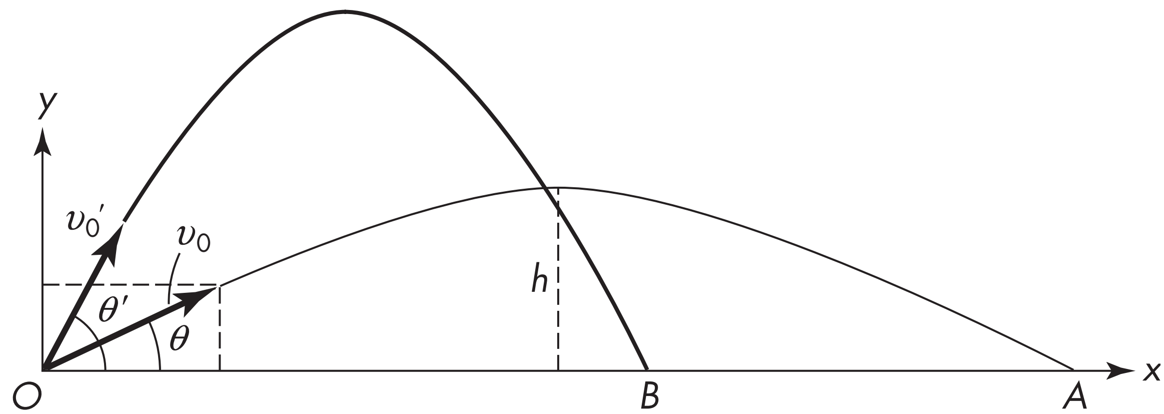
1. Which target got hit first?
   - Context of the textbook: Before Example 4.
2. Projectile range
   - A problem comparable to Example 7, except here the initial values given are v_{0x} and v_{0y}.
3. Gun and target
4. Gun and target on the moon
   - Questions 4.3 and 4.4 are supplementary problems for projectial motion.

Motion along a circular arc: These two questions are conventional problems in other engineering physics textbooks beyond the uniform circular motion discussed in this section. They may be introduced after Example 9 of Section 4.5.

5. Deceleration of a train
6. Motion of a simple pendulum

Relative velocity and the addition of velocities.
These questions may be used before Example 10 in Section 4.6.

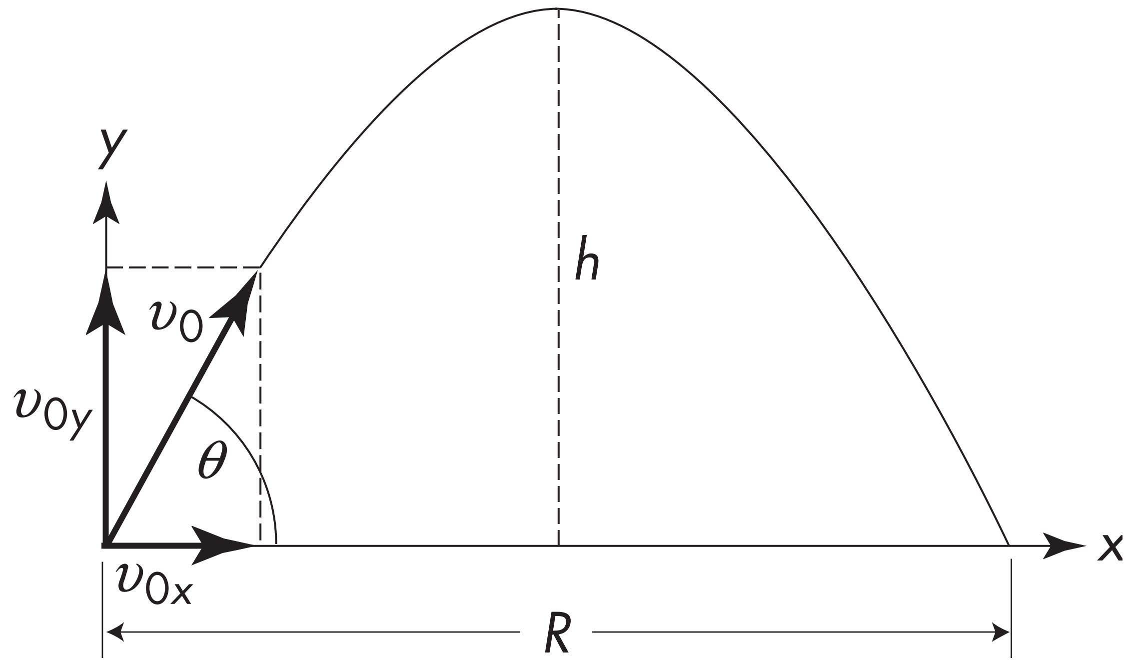
7. Definition of relative velocity
   - A basic conceptual question.
8. Crossing a river
   - A simplified version of Example 10.



Two projectiles are launched simultaneously and with the same launching speed from point O as shown $\mathbf{v}_{0'} = \mathbf{v}_0$. The first is launched at angle θ and hits target A in time t_A. The second is launched at a greater angle $\theta' > \theta$ and hits target B in time t_B. Compare the flight times for the two projectiles:

A	B	C
$t_B > t_A$	$t_B = t_A$	$t_B < t_A$

Explanation: Consider the projected motion along the vertical direction. The flight time for the first case is given by $t_{\text{flight}} = 2t_{\text{rise}} = 2v_{0y}/g$. For the second case, the y component of the initial velocity $v_{0y} = v_0 \sin \theta$ is replaced by $v'_{0y} = v'_0 \sin \theta'$, where $\theta' > \theta$ and $v_0 = v'_0$ are given. Therefore $v'_{0y} > v_{0y}$, which means that the flight of the second case takes longer. Answer = A.



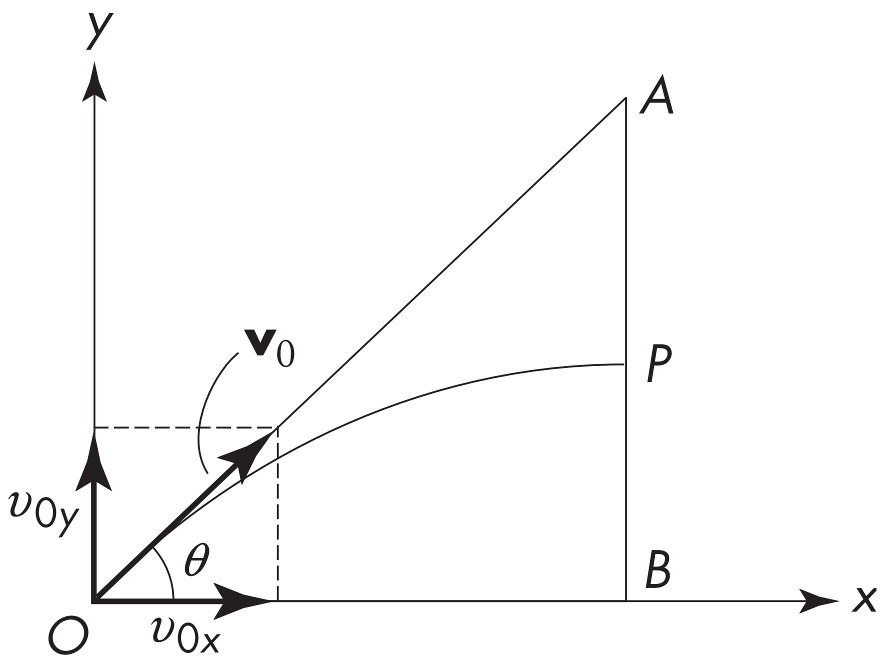
Given $v_{0x} = 20$ m/s and $v_{0y} = 10$ m/s and $g = 10$ m/s^2. Find R.
Choose one of the following:

A	$2v_0^2/g = 2(20^2 + 10^2)/10 = 100$ m
B	$v_{0x}v_{0y}/g = 20 \times 10/10 = 20$ m
C	$2v_{0x}v_{0y}/g = 2 \times 20 \times 10/10 = 40$ m

Hint: The rising time $t_{rise} = v_{0y}/g$, $v_0^2 = v_{0x}^2 + v_{0y}^2 = 500$ m^2.
Extra: Keep v_0 fixed and vary θ to find the maximum value of R.

Explanation: $R = v_{0x}\,(2t_{rise}) = 2v_{0x}v_{0y}/g$. Answer = C

Explanation—extra: $R = 2v_0^2 \sin\theta \cos\theta/g = v_0^2 \sin 2\theta/g$. So the maximum value of R when v_0^2 is fixed is $R = v_0^2/g = 50$ m.



Consider a gun at *O* aiming at a target located at point *A*. The gun fires at time $t = 0$, and at the same time the target begins to fall from rest. Let $v_{Ox} =$ 10 m/s and $v_{Oy} = 20$ m/s. Also, let $OB = 20$ m and $AB = 40$ m. Find the height *BP* as the bullet passes the vertical line *AB*:

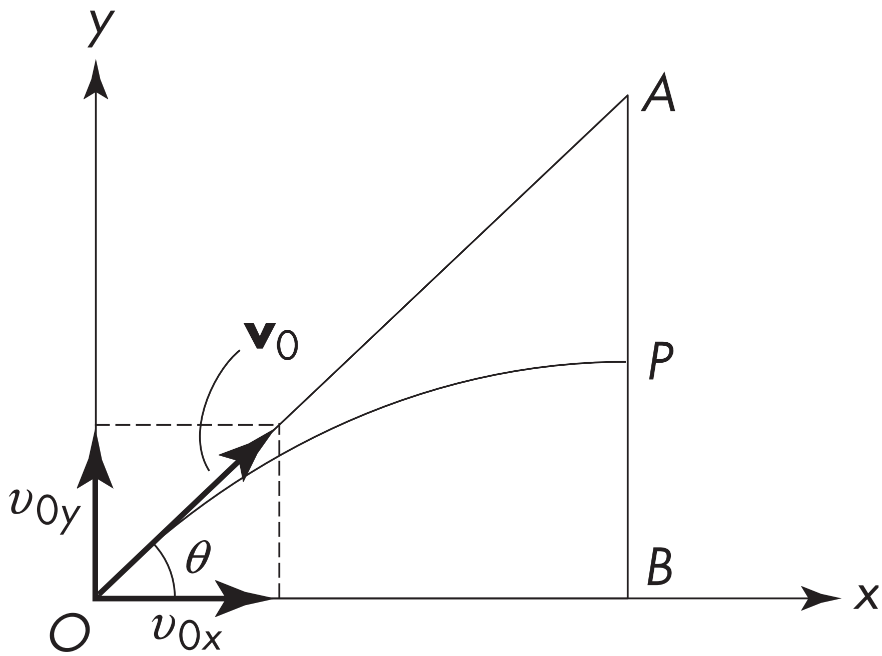
	A	B	C
BP (in m)	15	20	30

Hint: In the *x* direction $t_{OP} = OB/v_{ox} = 2$ s. To find *BP*, use $y = v_{Oy}t - gt^2/2$ for the motion of the bullet along the *y* direction. Assume $g = 10$ m/s^2.

Explanation: From point *O* to point *P*, the time that the bullet spends traveling vertically is the same as the time it spends traveling from O to P, $t_{BP} = t_{OP} = 2$ s. Use the equation of motion in the vertical direction: $BP = v_{Oy}t_{BP} - gt_{BP}^2/2 = (20)(2) - (10/2)(2)^2 = 20$ m. Answer = B.

An alternative method: *BP* may also be determined by considering the fall of the target. $AP = (1/2)gt_{BP}^2 = (10/2)(2)^2 = 20$ m. $BP = AB - AP =$ $40 - 20 = 20$ m.

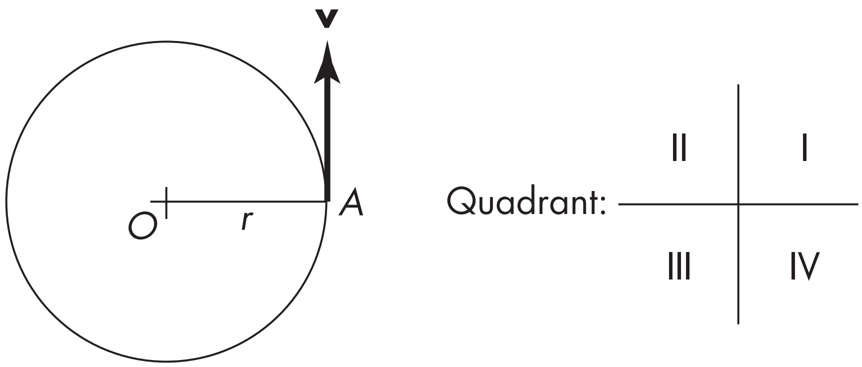
3. Gun and Target



Consider the "gun and target" setup of the previous problem, where the initial velocity vector $\mathbf{v}_0$, the length of the base line *OB*, and the height *AB* are kept fixed. Compare the height *BP*, when the experiment is carried out on Earth to the height *BP*′, when the experiment is done on the Moon:

A	B	C
$BP' < BP$	$BP' = BP$	$BP' > BP$

Explanation: Because v_{0x} and *OB* are the same for the two cases, the time of flight $t = OB/v_{0x}$ is also the same. During this time, on Earth the target falls by a distance $AP = (1/2)gt^2$. Because on the Moon the gravitational acceleration g' is less than g, $AP' < AP$. In turn, $BP' > BP$. Answer = C.



A train is moving along a circular track of radius $r = 100$ m. At point A, $v = |\mathbf{v}| = 10$ m/s.

It is slowing down with a tangential deceleration of a magnitude $a_{tangent} = |\mathbf{a}_{tangent}| = 1\ \text{m/s}^2$.

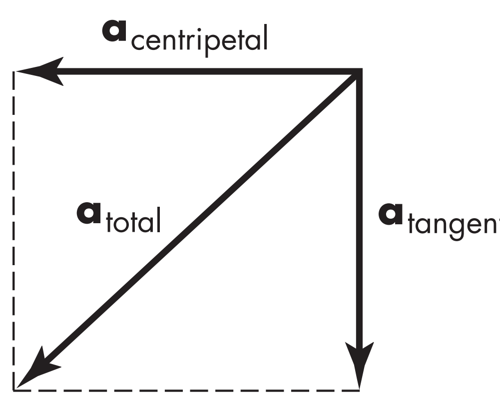
Sketch $\mathbf{a}_{total}$ at A. Which quadrant should it be in?

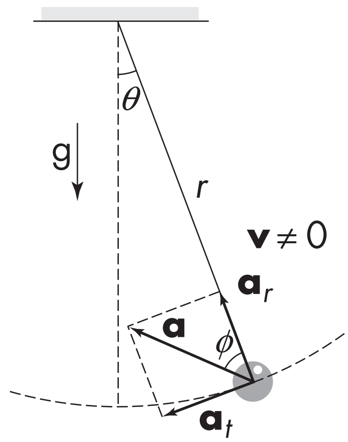
	A	B	C	D
Quadrant	I	II	III	IV

Extra: Find the magnitude $|\mathbf{a}_{total}|$.

Explanation: From the sketch at point A, $\mathbf{a}_{total}$ is in the third quadrant. Answer = C.

Explanation—extra: At point A, centripetal acceleration $\mathbf{a_c} = v^2/r = 10^2/100 = 1\ \text{m/s}^2$. So $a_{total} = \sqrt{a_c^2 + a^2{}_{tangent}} = \sqrt{1^2 + 1^2} = \sqrt{2}\ \text{m/s}^2$, or $\approx 1.4\ \text{m/s}^2$.





A simple pendulum consists of a string of length r and a ball attached to its end. When the string makes an angle θ with the vertical and the tangential velocity of the ball is pointing toward the vertical line, determine the corresponding tangential acceleration:

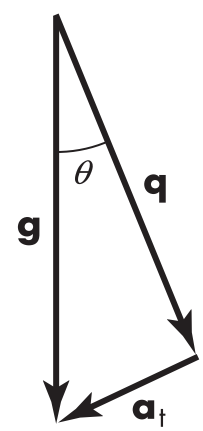
	A	B	C	D
$a_{tangent}$	$g \sin\theta$	$g \cos\theta$	$-g \sin\theta$	$-g \cos\theta$

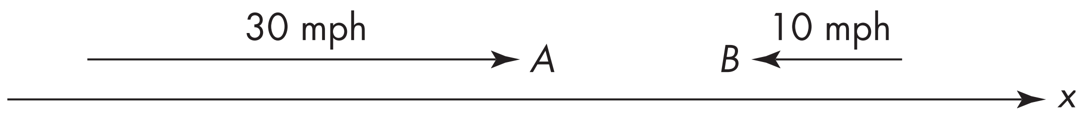
Hint: The angle θ is measured from the vertical line in a counterclockwise manner.

Extra: Compare θ with ϕ in the sketch.

Explanation: From the sketch, we see that the tangential acceleration is pointing toward the vertical line; thus it has a sign opposite to that of θ. Answer = C.

Explanation—extra: Notice in the top sketch, the centrepetal vector $\mathbf{a}_t = \mathbf{g}_r$ where $\mathbf{g}_r$ is shown in the second sketch. Since both vectors $\mathbf{a}$ and $\mathbf{g}$ are diagonals of two rectangles with same sides a_t and g_r, $\phi = \theta$. One can see this equality visually, if $\mathbf{a}_t$ is drawn to scale.





Car A travels at a speed of 30 mph to the right (positive x direction) and car B travels at 10 mph to the left. Consider the velocity $\mathbf{v}_{AB} = \mathbf{v}_{AB}\mathbf{i}$ to be the velocity of car A observed by the driver in car B (in other words, $\mathbf{v}_{AB}$ is the velocity of A relative to B). Given the x axis orientation as shown, find v_{AB} and v_{BA}.

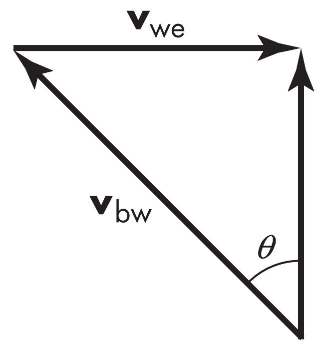
Choose one (in mph):

	A	B	C	D
v_{AB}	20	20	40	40
v_{BA}	20	−20	40	−40

Explanation: Notice that the driver of car B sees that car A is moving toward him or her—that is, along the positive x direction with a speed greater than $v_A = 30$ mph. The algebra involved is given here:

$v_{AB} = v_A - v_B = 30 - (-10) = 40$ mph.

$v_{BA} = v_B - v_A = -v_{AB} = -40$ mph. Answer = D.



The diagram here shows a boat attempting to cross a river. Assume that the boat's speed relative to the water $v_{bw} = 10$ m/s and that the current (the water's speed relative to the Earth) $v_{we} = 5$ m/s.

Find θ such that the boat crosses the river at a right angle to the bank:

	A	B	C
θ	30°	45°	60°

Extra: What is v_{be}, the boat's speed relative to the Earth?

Explanation: Because $\sin \theta = v_{we}/v_{bw} = 5/10 = 0.5$, $\theta = 30°$.

Explanation—extra: $v_{be} = v_{bw} \cos 30° \approx 8.7$ m/s.