chapter 6 Further Applications of Newton's Laws

Friction (Section 6.1)

1. Two blocks
2. Moving along the ceiling
3. Pulling at an angle
4. Sliding down an incline with friction
5. Blocks and pulley with friction

Spring force: Hooke's Law (Section 6.2)

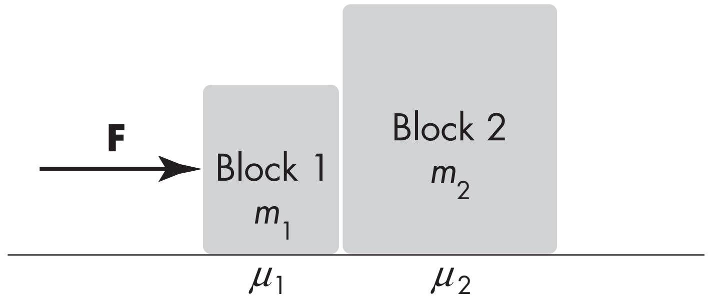
6. N identical springs in series

Uniform circular motion (Section 6.3)

7. Conic pendulum
8. Car on a curve

Additional conceptual exercises for circular motion and Section 6.3

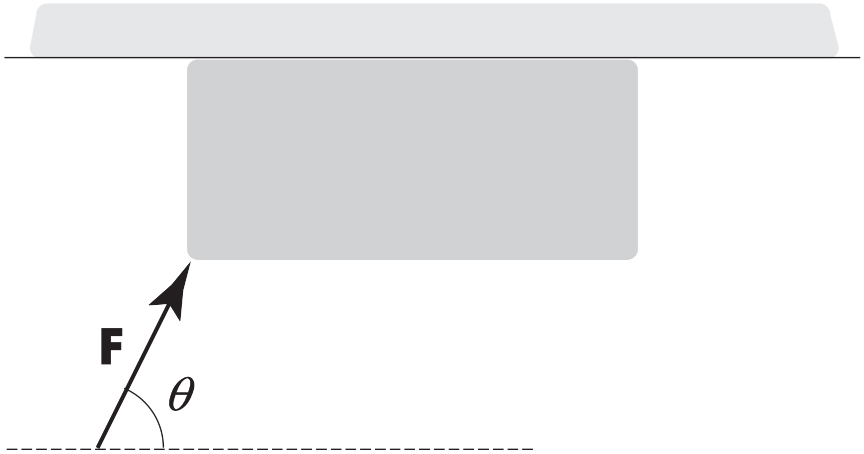
9. A Ferris wheel
10. Within a cyclone
11. A spinning toy



A two-block system experiences a horizontal applied force as shown here. Denote the force exerted on block 2 by block 1 as F_{21}. The coefficients of kinetic friction between block 1 and block 2 and the surface are μ_1 and μ_2 respectively. If the acceleration of the system is a, then the equation of motion given by Newton's Second Law for block 2 is given by which of the following?

A	$F - \mu_1 m_1 g - \mu_2 m_2 g = m_2 a$
B	$F_{21} - \mu_2 m_2 g = m_2 a$
C	$F + F_{21} - \mu_2 m_2 g = m_2 a$

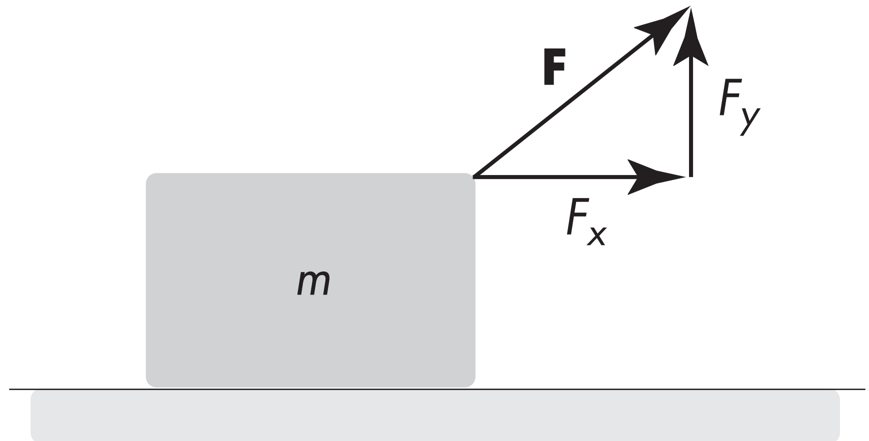
Explanation: The net force on block 2 is $F_{21} - \mu_2 m_2 g$. Applying Newton's Second Law, $F_{net} = ma$, leads to Answer = B.



The force **F** exerted on the block pushes the block against the ceiling and at the same time accelerates the block to the right. Find the force of the kinetic friction between the block and the ceiling:

	A	B	C	D
f_k	$\mu_k mg$	$\mu_k F \sin\theta$	$\mu_k (F \sin\theta - mg)$	$\mu_k (F \sin\theta + mg)$

Explanation: The net normal force between the block and the ceiling is $N = F \sin\theta - mg$. So the force of kinetic friction is $f_k = \mu_k N = \mu_k (F \sin\theta - mg)$. Answer = C.



The diagram here shows a block of mass $m = 1$ kg being pulled by force **F**, where $F_x = 8$ N and $F_y = 6$ N. The block is on a horizontal surface, and the coefficients of friction between the block and the surface are $\mu_s = 0.7$ and $\mu_k = 0.5$. Use this information to find the force of friction f on the block:

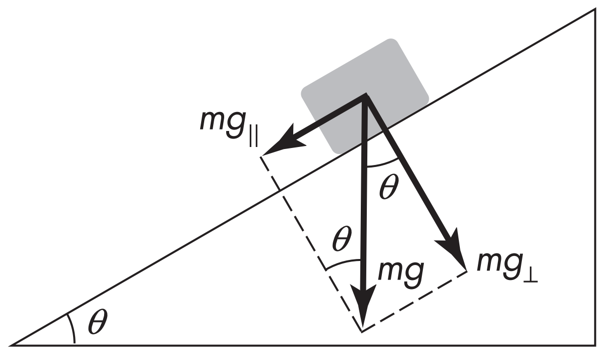
A	$f = F_x = 8$ N
B	$f = \mu_s\,(mg - F_y) = 0.7 \times 4 = 2.8$ N
C	$f = \mu_k mg = 0.5 \times 10 = 5$ N
D	$f = \mu_k\,(mg - F_y) = 0.5 \times 4 = 2$ N

Hint: $N = mg - F_y = 1 \times 10 - 6 = 4$ N, $f_{s,max} = \mu_s\,N = 0.7 \times 4 = 2.8$ N. Should the friction be $f_s = F_x$ or $f_k = \mu_k\,N$?

Extra: Determine the acceleration of the block.

Explanation: To determine if the block will be moving, compare the horizontal applied force $F_x = 8$ N to the maximum static friction, $f_{s,max} = 2.8$ N given the hint. Because the applied force is greater, the block will be moving to the right, and the friction force will be due to the kinetic friction. Equations $f_k = \mu_k\,N$ and $N = mg - F_y$ lead to Answer D.

Explanation—extra: Acceleration is given by $a = F_{net}/m = (F_x - f)/m = (8 - 2)/1 = 6\ \text{m/s}^2$.



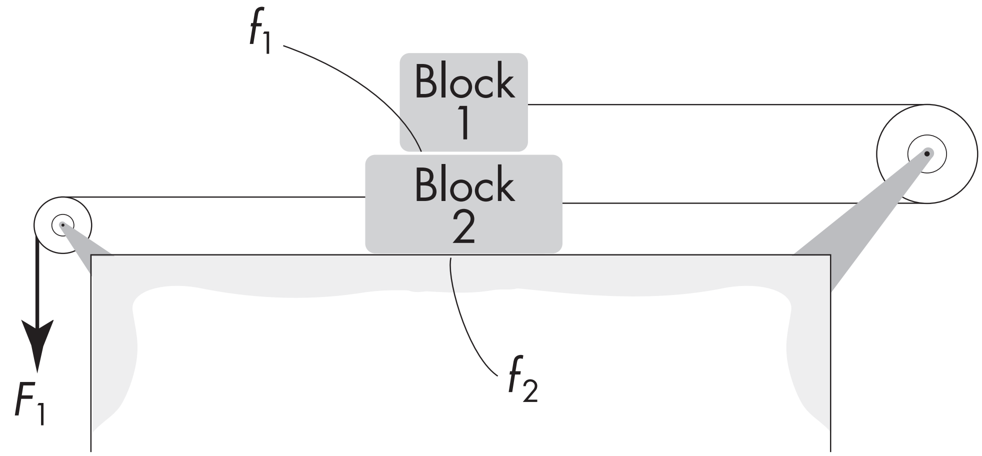
The diagram here shows a block of mass $m = 1$ kg sliding down a fixed incline. If the coefficient of static friction between the block and the incline $\mu_k = 0.8$ and the angle of inclination $\theta = 60°$, find the friction force f on the block:

A	$f = \mu_s mg_{\|} = \mu_s mg \sin 60° = 8.7$ N
B	$f = \mu_s mg_\perp = \mu_s mg \cos 60° = 5.0$ N
C	$f = \mu_k mg_\perp = \mu_k mg \cos 60° = 4.0$ N

Extra: Find the acceleration.

Explanation: Along the incline, the downhill gravitational force is $F = mg_n = 8.7N$, and the maximum opposing static-force is $f_{s,max} = \mu_s N$, where N is the magnitude of the normal force between the incline and the block, then $N = mg_\perp$ and $f_{s,max} = \mu_s mg = 5.0N$. Answer = C.

Explanation—extra: To find the acceleration, we use the equation of motion: $mg_{||} - f_k = ma$, $8.7 - 4.0 = 4.7 = 1 \times a$. Solving for a gives $a = 4.7$ m/s^2.



Consider the system shown in which the pulleys are frictionless and the applied force F_1 is large enough so that both blocks are moving. Let f_1 be the friction force between blocks 1 and 2. Also let f_2 and ${}_{2k}$ respectively represent the friction force and the coefficient of kinetic friction between block 2 and the table. Determine the correct pair of equations from the following:

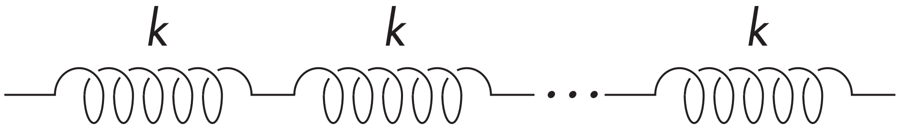
I: $F_1 - 2f_1 - f_2 = (m_1 + m_2)a$
I′: $F_1 - f_1 - f_2 = (m_1 + m_2)a$
II: $f_2 = {}_k m_2 g$
II′: $f_2 = {}_k(m_1 + m_2)g$

A	B	C	D
I and II	I′ and II	I and II′	I′ and II′

Extra: Starting from rest, what is the minimum force F_1 in terms of the opposing maximum static friction force, $f_{15,max}$ and $f_{25,max}$ such that the blocks will start to move?

Explanation: The frictions shown are directed in such a way that they oppose F_1. f_2 and f_1 act on m_2. f_1 acts on m_1. Using Newton's Second Law gives $F_1 - (f_2 + f_1 + f_1) = (m_1 + m_2)a$. The normal force associated with f_2 is $(m_1 + m_2)g$. So the correct choice is I and II′. Answer = C.

Explanation—extra: F_1 must be slightly greater than the opposing maximum static force before the blocks will move: $F_1 > 2f_{1s,max} + f_{2s,max}$.



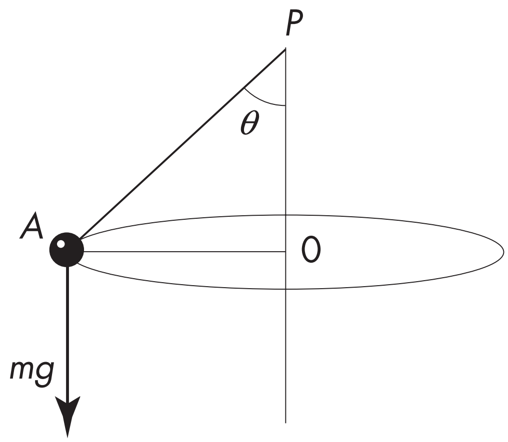
Consider a spring system that consists of N identical springs each with a spring constant k connected in series. Determine the effective spring constant of this system, k_{eff}. Neglect the mass of the springs.

Choose one answer:

A	B	C	D
$N\,k$	$N\,k/2$	$2\,k/N$	k/N

Hint: Consider a force F that is pulling at the end of the spring system. Neglecting the mass of the springs, this force should be transmitted undiminished throughout the spring system: $F_i = F$, where $i = 1, 2, \ldots, N$.

Explanation: If the spring system is stretched by the force F for a distance x, then the effective spring constant of the spring system k_{eff} is by definition defined by $F = -k_{\text{eff}}x$. The equality of the force applied to each of the springs implies that each spring should be stretched by an equal amount—that is, by x/N. In other words, the force on the ith spring is given by $F_i = -kx_i = -k\,x/N = F = -k_{\text{eff}}\,x$. It follows that $k_{\text{eff}} = k/N$. Answer = D.



Consider a conic pendulum with mass m and the angle θ. Define the following:

I: $T \sin \theta = F_{\text{centripetal}}$ I′: $T \cos \theta = F_{\text{centripetal}}$
II: $T \sin \theta = mg$ II′: $T \cos \theta = mg$

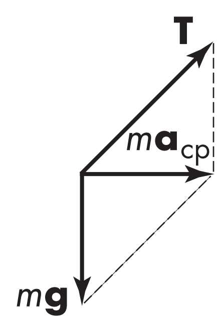
Which is the correct equation along the x direction?

A	B	C	D
I	II	I′	II′

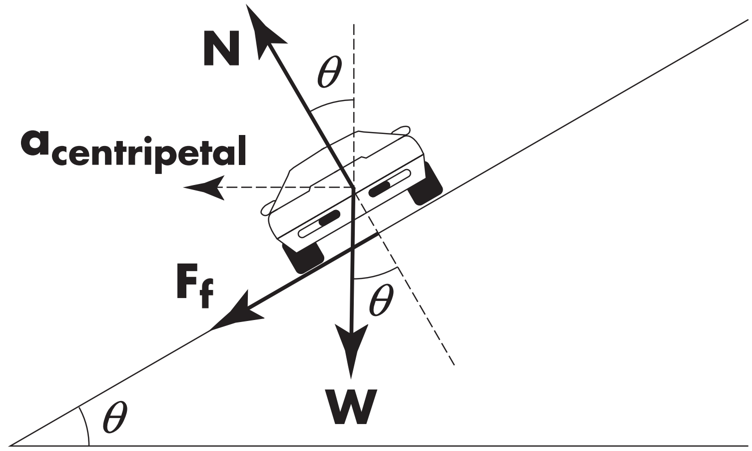
Hint: Draw a "free-body" diagram showing the forces acting on the mass. These forces are the gravitational force and the tension from the pendulum string.

Extra: Choose the correct equation along the y direction.

Explanation: The "free-body" diagram is where the centripetal force $\boldsymbol{F}_{CP}$ = ma cp is directed horizontally toward the center of the circle of rotation. From the sketch, for the x equation, I is correct. Answer = A.



Explanation—extra: From the sketch, for the y equation, II′ is correct.



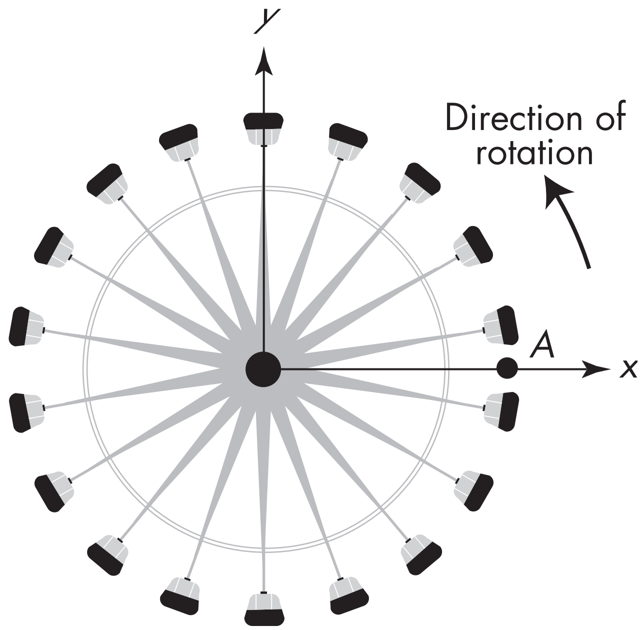
A car rounds a slippery curve maintaining the same height. Its motion is along a circular path in a horizontal plane. If there is friction between the tires and the road and the friction is pointing down the slope, how will you compare the speed of the car v and the optimal speed for the banked curve v_{op}?

Choose one of the following:

A	B	C
$v > v_{op}$	$v = v_{op}$	$v < v_{op}$

Explanation: Because the friction force is pointing downhill, this friction must be due to the tendency for the car to skid upward. This tendency is due to the fact that the speed of the car is greater than the optimal speed: Answer = A.

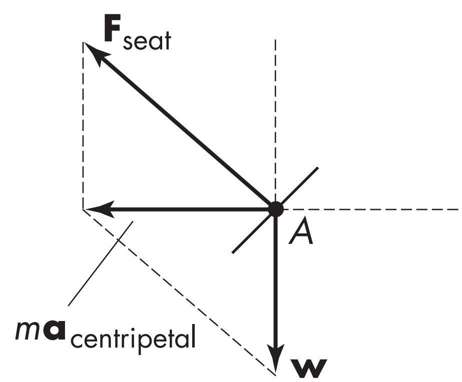
8. Car on a Curve



Consider the setup of a Ferris wheel in an amusement park. The wheel is turning counterclockwise. Not all seats are aligned horizontally (parallel to the *x* axis). Instead the angle of each seat changes as it travels around the Ferris wheel. Determine the orientation of a seat as it passes by point *A*:

A	Parallel to the *x* axis
B	In the first/third quadrants
C	Parallel to the *y* axis
D	In the second/fourth quadrants

Explanation: As a chair rises past point *A*, apply Newton's Second Law to the rider. As shown in the "free-body" diagram here, $\mathbf{F}_{seat} + \mathbf{w} = m\mathbf{a}_{centripetal}$. The force exerted by the seat on the rider, $\mathbf{F}_{seat}$, is perpendicular to the seat. Because $\mathbf{a}_{centripetal}$ is oriented to the left at point *A*, the seat must be oriented in the first/third quadrants. Answer = B.

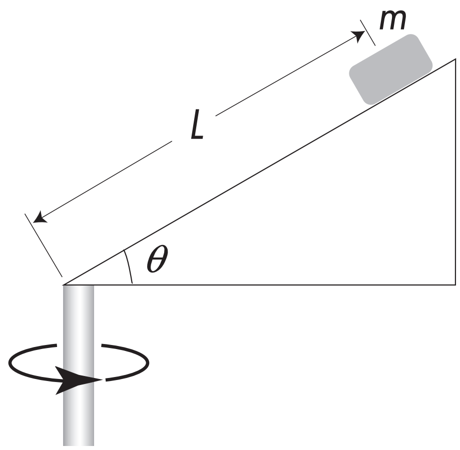


A cyclone consists of a large vertical cylinder that spins about its axis fast enough so that any person inside will be held up against the wall. Assume that when the speed is ω_1, there is an upward friction f_1 that holds the person up against the wall. How does the new friction f_2 compare to f_1 when the speed is doubled ($\omega_2 = 2\omega_1$)?

A	B	C
$f_2 < f_1$	$f_2 = f_1$	$f_2 > f_1$

Explanation: For the person to be held up against the wall, the net vertical force on the person is 0. So $f_1 = mg$. $f_2 = f_1 = mg$. In other words, even though the cylinder spins faster, there is no increase in the upward friction. Answer = B.

10. Within a Cyclone



A child's toy consists of a wedge with an angle θ. The wedge is spinning at a constant speed by rotating a rod that is firmly attached to one end. On the wedge, the block can freely slide along the slope. The wedge has enough traction so that the block will not slide along the tangential direction of rotation. When the wedge has an angular velocity ω, equilibrium position occurs at L. As ω increases, what is the corresponding new equilibrium length L'?

A	B	C
$L' > L$	$L' = L$	$L' < L$

Hint: The centripetal force is $F_{CP} = \frac{mv^2}{r} = m\omega^2 r$, where r is the perpendicular distance between m and the axis of rotation.

Explanation: At equilibrium the normal force N on m satisfies these relations: (1) $N \cos\theta = mg$ and (2) $N \sin\theta = F_{CP} = m\omega^2 r = m\omega^2 L \cos\theta$. See sketch below. (1) implies that N is independent of ω. (2) gives $N \tan\theta = m\omega^2 L$. Because the left side of the last equation is constant, as ω increases, the new equilibrium length along the ramp must decrease to keep $\omega^2 L$ fixed. Answer = C.