chapter 7 Work and Energy

Work

Work in one dimension: ***(Section 7.1, part 1)***

The following two problems involve pulleys and may be used after Example 2 in Section 7.1:

1. Atwood machine
2. Raising m by a distance Δx

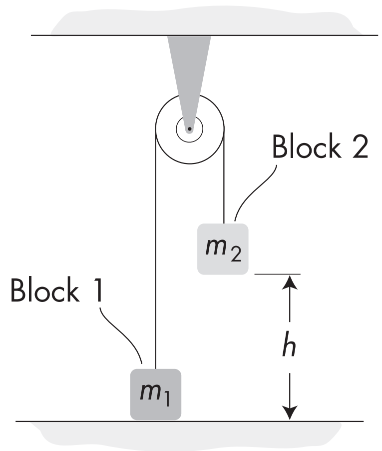
Work in two dimensions: ***(Section 7.1, part 2)***

3. Work against gravity
   - Context in the textbook: After Example 3 in Section 7.1.

Work–energy and gravitational potential energy (Section 7.4)

Exercises to be used in conjunction with Section 7.4:

4. Sliding down an incline
5. Comparing final kinetic energies
6. Stopped pendulum
7. Sliding down a dome: The “stay-on” condition
8. Sliding down a dome: Equations of motion

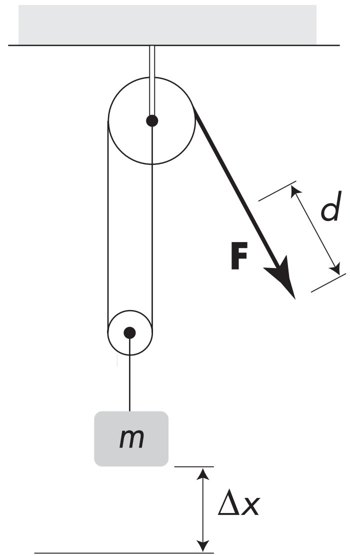


Two blocks with masses m_1 and m_2 are connected by a light string passing over a light frictionless pulley. Assume $m_2 > m_1$. Determine the potential energy released by the system as block 2 starts from rest and falls by $h/2$:

	A	B	C
Potential energy released	$m_2gh/2$	$(m_2 - m_1)gh/2$	$m_1gh/2$

Hint: Remember that as block 2 descends by $h/2$, block 1 rises correspondingly by a height $h/2$.

Explanation: When block 2 falls by a distance $h/2$, block 1 goes up by $h/2$. The potential energy lost by block 2 is $m_2gh/2$. The potential energy gained by block 1 is $m_1gh/2$. So the net potential energy lost is $(m_2 - m_1)gh/2$. Answer = B.



Consider the mass–pulley system shown. Determine the distance d covered by the force F as it lifts the block by a height Δx:

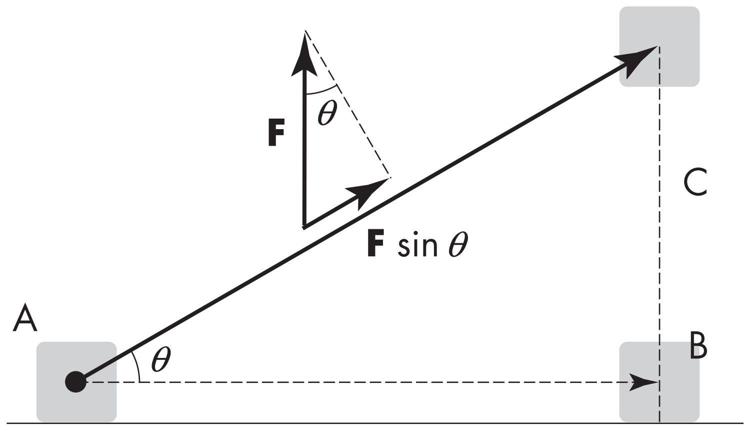
	A	B	C
Distance of F	$d = \Delta x/2$	$d = \Delta x$	$d = 2\Delta x$

Explanation: As the block is lifted by height Δx, the length of each of the two strings supporting the moving pulley will be reduced by Δx, so $d = 2\Delta x$. Answer = C.

Comment: Notice that conservation of energy implies that the increase of the potential energy equals the input mechanical energy: $mg\Delta x = \mathrm{F}d = 2\Delta x\,\mathrm{F}$. Solving for F gives $\mathrm{F} = mg/2$. In other words, the applied force required is half of the weight.

2. Raising m by a Distance Δx

Consider the movement of a block of mass $m = 1$ kg in two phases. In the first phase it moves horizontally 4 m from A to B. In the second phase it goes from B to C, a vertical distance of 3 m. Find the work done on the block against the force of gravity for each phase of its journey:



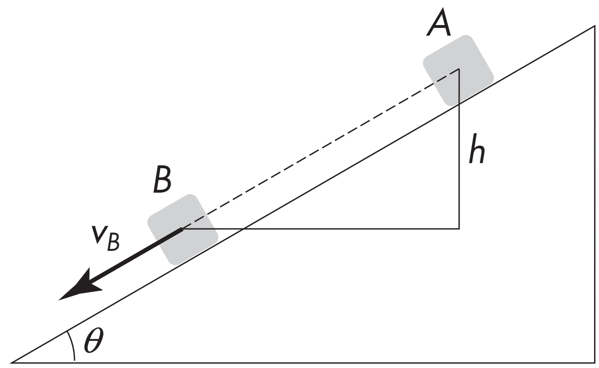
	W_{AB}	W_{BC}
A	40 J	0 J
B	0 J	30 J
C	40 J	30 J

Hint: Work is the product of force times displacement parallel to the force.

Extra: Consider moving the block directly from A to C along a frictionless inclined plane. Determine the work done, W_{AC}.

Explanation: From A to B there is no displacement parallel to gravitation force. So $W_{AB} = mg \times (0) = 0$ J. $W_{BC} = mg \times 3 = 30$ J. Answer = B.

Explanation—extra: Along the incline $W_{AC} = mg \sin\theta \times AC = mg \times BC = 10 \times 3 = 30$ J, where $mg \sin\theta$ is the lifting force along the inclined plane. This illustrates that the work done against a conservative force (that is, gravity) is path independent: The work along the two paths $A \rightarrow B \rightarrow C$ and $A \rightarrow C$ is the same.



Consider a block of mass $m = 1$ kg sliding down a frictionless inclined plane from rest at A. Given $h = 20$ m, $\theta = 30°$, and $g = 10$ m/s^2, find v_B, the speed of the block as it passes through point B:

A	$v_B = \sqrt{2gh} = 20$ m/s
B	$v_B = \sqrt{gh} = 10\sqrt{2}$ m/s
C	$v_B = \sqrt{gh/2} = 10$ m/s

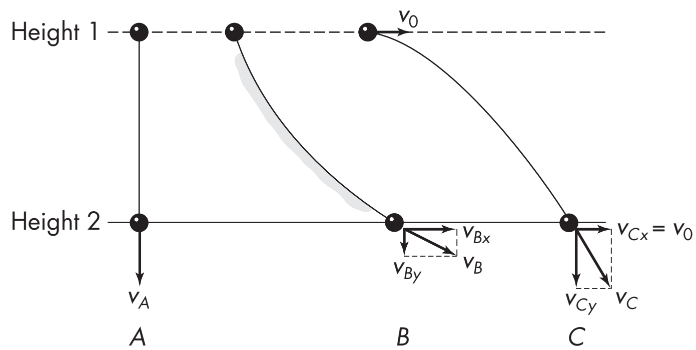
Hint: Use the conservation of energy equation $K_f - K_i = U_i - U_f$. (see Eq. (7.32)), where $K_i = K_A = 0$ and $U_i - U_f = U_A - U_B = mgh$.

Extra: Determine the kinetic energy in joules at B.

Explanation: $K_f = K_B = mv_B{}^2/2$. The conservation of energy equation implies that $mv_B{}^2/2 = mgh$, that is, $v_B = \sqrt{2gh}$. Answer = A.

Explanation—extra: $K_B = mv_B{}^2/2 = mgh = 1 \times 10 \times 20 = 200$ J.

Consider the three possible paths marked A, B, and C each for a given mass traveling from height 1 to height 2. A represents a free fall from rest. B represents starting from rest and sliding down a curved slide without friction. C represents a projectile with an initial horizontal speed of v_0 as shown.



Which set of relationships is correct for the speeds of the mass at height 2?

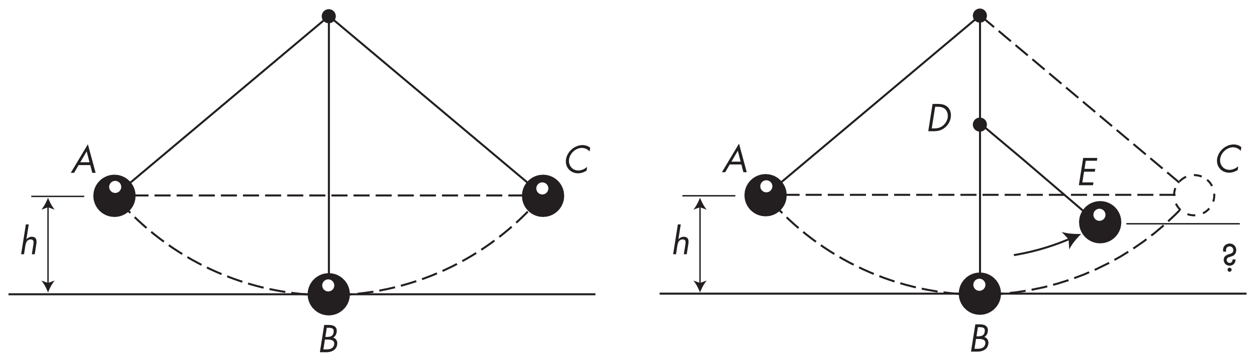
A	$v_A = v_{By} = v_{Cy}$
B	$v_A = v_B < v_C$
C	$v_A < v_B < v_C$

Hint: The frictionless surface that constrains the motion for path B neither creates nor dissipates energy. So $K_f - K_i = U_f - U_i$.

Extra: Compare the time taken to reach the ground for these three cases.

Explanation: For A and B, the initial kinetic energy K_i is 0. Conservation of energy implies $K_f = U_i - U_f = mgh$. In turn, the final kinetic energies are equal: $K_A = K_B$. Because the masses are the same, the final speeds are equal too: $v_A = v_B$. For C, $U_i - U_f$ is still equal to mgh; and because $K_i > 0$, conservation of energy implies that $K_C = U_i - U_f + K_i > K_A$. Answer = B.

Explanation—extra: The vertical motion in C is the same as that in A, so the trip time $t_A = t_C$. Now compare A and B. Because the distance covered in B is longer than the distance covered in A, it takes longer to reach the ground for B than for A.

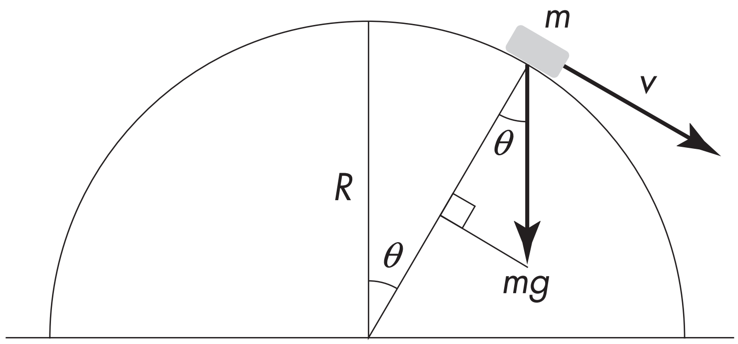


Consider the simple pendulum shown in which the friction force is negligible. The bob is oscillating between the two extreme points *A* and *C*, which have the same height *h*. Now place a stopper at *D*, located some distance below the initial pivot point. As the bob starts swinging to the right, it hits the stopper and pivots about point *D*. Denote the new extreme point by *E* as shown. The height at *E* is which of the following?

1	Greater than *h*
2	Equal to *h*
3	Less than *h*

Hint: Use the conservation of energy equation: $U + K =$ constant. This is valid at any point along the path of motion, with or without the stopper.

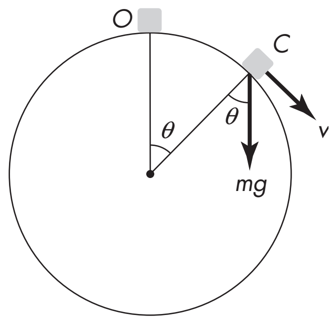
Explanation: Conservation of energy implies that $U_A + K_A = U_B + K_B$, or $K_B = U_A - U_B = mgh$, because $K_A = 0$. It also implies that $U_B + K_B = U_E + K_E$. Because *E* is the new extreme point, $K_E = 0$, the potential difference between *E* and *B* is $U_E - U_B = K_B = mgh = U_A - U_B$. In other words, point *E* has the same height as *A*. Answer = B.



A small box of mass m is initially at the top of a hemisphere with a radius R. Starting from rest, the box slides without friction down along the surface of the hemisphere. At any θ, the centripetal force is $F_c = mv^2/r$. What is the condition for the block to stay on the surface?

A	B	C	D
$mg \cos\theta < F_c$	$mg \sin\theta < F_c$	$mg \cos\theta > F_c$	$mg \sin\theta > F_c$

Explanation: When the radial component of the weight is larger than the centripetal force, there is a net force directed radially inward that presses the block against the surface. This is the condition for the block to stay on the surface. Answer = C.



Consider a block starting at rest from point O, sliding down the surface of a smooth sphere (friction is negligible) and flying off at point C. Here two principles are at work:

I. Conservation of energy: $U_O + K_O = U_C + K_C$.

II. At the flying-off point the surface exerts no force on the block. In other words, at point C the normal force $N = 0$. Define the following relations:

For I: $mgR(1 - \cos\theta) = mv^2/2$ Ia
$mgR(1 - \sin\theta) = mv^2/2$ Ib

For II: $mg\cos\theta = mv^2/R$ IIa
$mg\sin\theta = mv^2/R$ IIb

Determine the correct pair of relations:

	A	B	C	D
For I	Ia	Ia	Ib	Ib
For II	IIa	IIb	IIa	IIb

Explanation: For I, the vertical distance between O and C is $R(1 - \cos\theta)$, so choice Ia is correct. For II, the radial component of mg is $mg\cos\theta$. Here the choice IIa is correct. Combining the two leads to Answer A.

Comment: Based on the two relations at C, we may solve for $\cos\theta$. Verify that $\cos\theta = 2/3$.