chapter 8 Conservation of Energy

Potential energy of a conservative force (Section 8.1)

1. Mass–spring system
   - Context in the textbook: Before Example 1 in Section 8.1.
2. Conservative force: Path integrals
   - Context in the textbook: Section 2.1 considers conservative force in one dimension. This example verifies the path-independent property of a conservative force in two dimensions. It may be used at the end of Section 8.1.

Potential energy due to gravity plus spring force (Section 8.2)

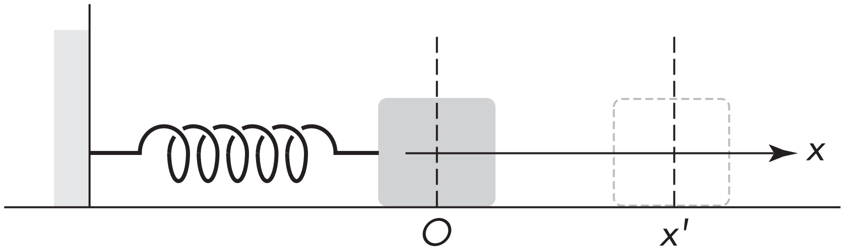
The following exercises may be used before Example 4 in Section 8.2:

3. Spring toy: Maximum height
4. Spring toy: Special points of U
5. Releasing a compressed spring
6. Spring–pulley system with friction

Power (Section 8.5)

Two exercises before Example 9 of Section 8.5:

7. Climbing up a hill
8. Maximum speed on a flat road



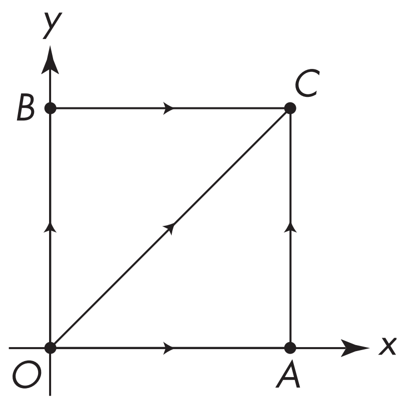
Consider a mass–spring system. The spring force asserted on the block is given by Hooke's Law: $F_{spring} = -k\Delta x$. Denote the potential energy of the spring by $U(x)$. At $x = 0$ the spring is relaxed: $U(0) = 0$. $U(x)$ is the work done by the stretching force in going from $x = O$ to $x = x'$, which is given by which of the following?

A	B	C	D
$-kx^2$	kx^2	$\int_0^{x'} kxdx$	$-\int_0^{x} kxdx$

Extra: Release the mass from rest at $x = x'$. Determine the kinetic energy at $x = 0$ in terms of x', k, m. *Hint:* Not all the quantities here need to be used.

Explanation: When the spring is pulled to the right, both the pulling force and the displacement are in the same direction. The pulling force is opposite to the spring force. The latter obeys Hooke's Law: $F_{pull} = -F_{spring} = k\Delta x$. So work done in stretching the spring from $x = O$ to $x = x'$ is given by Answer C.

Extra—explanation: Conservation of energy implies that the kinetic energy at O is the potential energy released as the mass moves from $x = x'$ to $x = 0$, which is $kx'^2/2$.

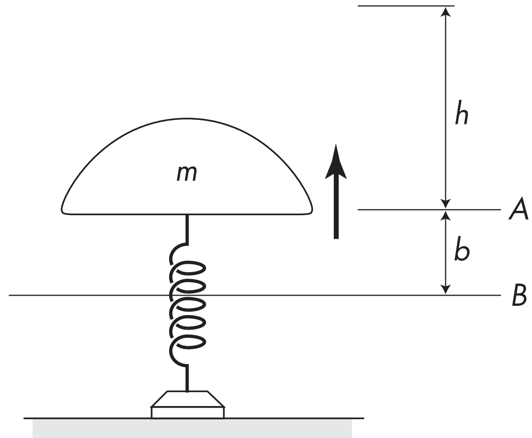


A particle is under the influence of a force: $\mathbf{F} = 2xy\mathbf{i} + x^2\mathbf{j}$. Determine the relationship between work done by this force along two different paths: $O \rightarrow A \rightarrow C$ and $O \rightarrow B \rightarrow C$. At C, $x_C = x_A = 1$ and $y_C = y_B = 1$. Choose one of the following solutions:

A	$W_{OA} + W_{AC} > W_{OB} + W_{BC}$
B	$W_{OA} + W_{AC} = W_{OB} + W_{BC}$
C	$W_{OA} + W_{AC} < W_{OB} + W_{BC}$

Explanation: Notice that along each segment either the x integral or the y integral (not both) is involved.
$W_{OA} = \int_{OA} 2xydx|_{OA} = 0$ because $y = 0$ along OA.
$W_{AC} = \int_{AC} x^2dy|_{AC} = 1$. Here $x = 1$ along AC was used.
$W_{OB} = \int_{OB} x^2dy|_{OB} = 0$ because $x = 0$ along OB.
$W_{BC} = \int_{BC} 2xydx|_{BC} = 1$. Here $y = 1$ along BC was used.
Put it all together: $W_{OC} = W_{OA} + W_{AC} = W_{OB} + W_{BC} = 1$. Answer = B.

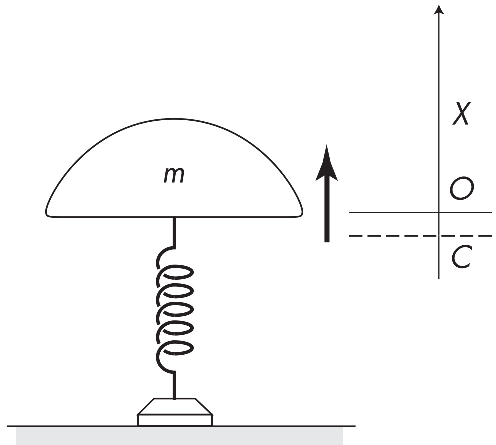
Digression: The path independence of W_{OC} is a necessary condition for $\mathbf{F}$ to be conservative. The extension of Eq. (8.21) to two dimensions is to replace the derivative there by partial derivatives: $F_x = -\partial U/\partial x$, and $F_y = -\partial U/\partial y$. We can easily verify that $U = -x^2y$ leads to the expressions of F_x and F_y given here. The extension of Eq. (8.20) leads to (as expected) $W_{OC} = -(U_C - U_O) = -([-x^2y]_C - [-x^2y]_O) = -(-(1) + 0) = 1$, in agreement with the results here.



A toy has a plastic top of mass m attached to a spring with a spring constant k. The top is pressed from position A—the relaxed position of the spring when no mass is attached to it—to position B. The vertical distance between A and B is b. After the plastic top is released from position B, how high will the top rise? Let h be the height measured from A. Choose one answer:

	A	B	C
h	$\dfrac{kb^2}{(2mg) - b}$	$\dfrac{kb^2}{(2mg)} - b$	$\dfrac{kb^2}{(2mg)}$

Explanation: Conservation of energy implies that $\left(\dfrac{1}{2}\right)kb^2 = mg(h + b)$. Solving for h leads to Answer B.



Consider the spring toy of the previous question, Question 3. Let the equilibrium position of the spring in the absence of the top be at the origin *O*; the equilibrium position in the presence of the top is at *C*. Find the magnitude *OC*:

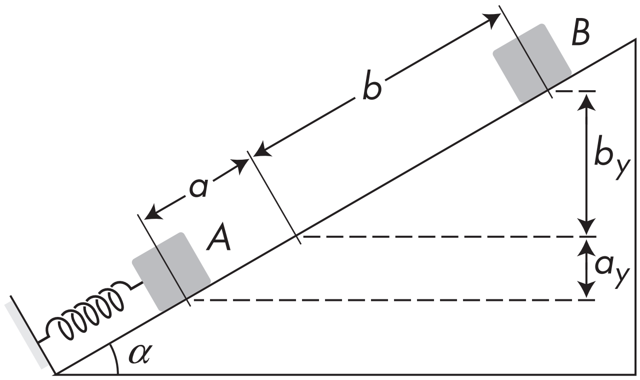
	A	B	C
OC	mg/k	$2mg/k$	$mg/2k$

Hint: The potential function is given by $U = kx^2/2 + mgx$.

Extra: Determine the two x values where $U = 0$.

Explanation: Based on the potential function given, the corresponding force asserted on the top mass is given by $F = -\partial U/\partial x = -kx - mg$. The equilibrium occurs at $x = -mg/k$. In other words, the magnitude $OC = mg/k$. Answer = A.

Extra—explanation: Solving the equation $U = kx^2/2 + mgx = 0$ gives $x = 0$ and $x = -2mg/k$. Notice the equilibrium point (where the net force from the weight plus the spring force is zero) is the midpoint between the two points where $U = 0$. A similar situation occurs as described in Fig. 8.12 of Section 8.2.



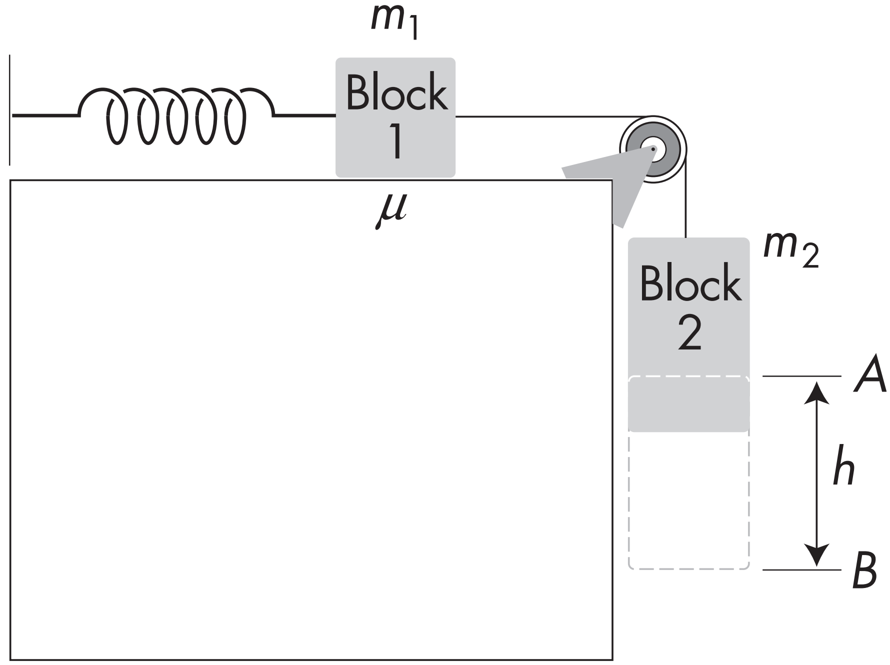
Consider the uphill motion of this block after the release of a compressed spring at *A*. The block stops at *B*. Write the work–energy theorem that relates distance $AB = a + b$ to the other quantities listed here:

Notations:	Mass of the block	m
	Spring constant	k
	Coefficient of kinetic friction	μ_k
	Initial compression	a
	Angle of the incline	α

A	$ka^2/2 = mg(a + b)(\sin\alpha + \mu_k \cos\alpha)$
B	$ka^2/2 = mg(a + b)(\sin\alpha + \mu_k \sin\alpha)$
C	$ka^2/2 = mg(a + b)(\cos\alpha + \mu_k \cos\alpha)$
D	$ka^2/2 = mg(a + b)(\cos\alpha + \mu_k \sin\alpha)$

Hint: $W^{ext}{}_{AB} = (K_B - K_A) + (U^g{}_B - U^g{}_A) + (U^{sp}{}_B - U^{sp}{}_A) + W^{dis}{}_{AB}$

Explanation: There is no external force involved, so $W^{ext}{}_{AB} = 0$. The block is at rest at *A* and at *B*: $K_B = K_A = 0$. When the block is at *B*, the spring is relaxed: $U^{sp}{}_B = 0$. At *A* the spring is compressed by *a*, so $U^{sp}{}_A = ka^2/2$. The gravitational potential energy $U^g{}_B - U^g{}_A = g(a_y + b_y) = mg(a + b)\sin\alpha$. The dissipative energy (due to frictional force *f*) is the only contribution to $W^{dis}{}_{AB} = fs = \mu_k mg_\perp(a + b) = \mu_k mg(a + b)\cos\alpha$. Putting this all together and rearrangement leads to the equation of Answer A.

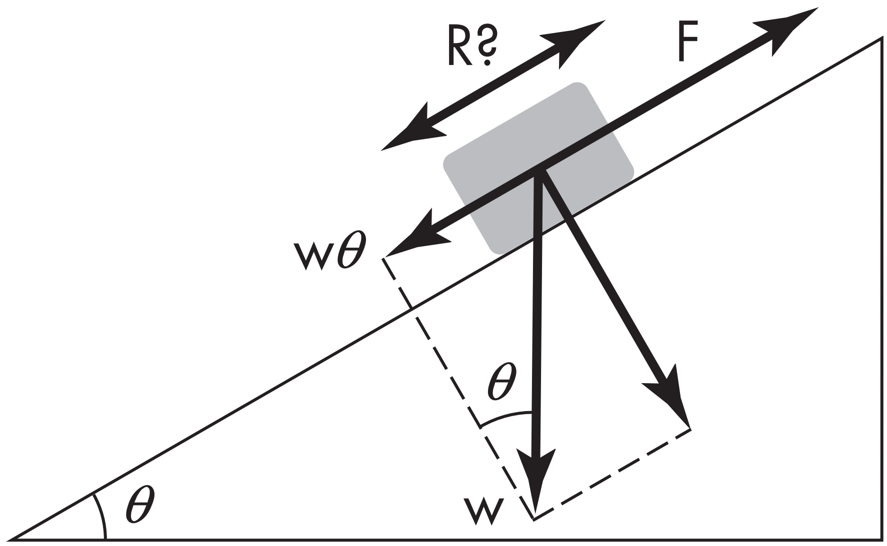


Consider the spring–pulley system shown here. Assume that the coefficient of friction μ, the spring constant k, the mass m_1, and the mass m_2 are all given. When the bottom of block 2 is at position A, the spring is in the relaxed state. When block 2 is released from rest at this state, the system moves clockwise. This motion continues until block 2 comes to rest at position B. The work–energy relation from A to B leads to which of the following?

A	$m_1gh = kh^2/2 + \mu m_1gh$
B	$m_2gh = kh^2/2 + \mu m_1gh$
C	$(m_1 + m_2)gh = kh^2/2 + \mu m_1gh$

Hint: In the absence of external force, the energy equation gives $0 = (U^g{}_B - U^g{}_A) + (U^{sp}{}_B - U^{sp}{}_A) + W^{dis}{}_{AB}$ where U^g and U^{sp} are potential energies due to gravity and spring, respectively; $W^{dis}{}_{AB}$ is the dissipative work done by friction; and the kinetic energies are not included because the blocks start and stop at rest.

Explanation: $(U^g{}_B - U^g{}_A) = m_2gh$, $(U^{sp}{}_B - U^{sp}{}_A) = kh^2/2$, $W^{dis}{}_{AB} = \mu m_1gh$. Putting these terms together and rearranging them leads to the expression of Answer B.



A car is climbing up a hill with a constant speed v. It experiences air resistance R = 1000 N. The weight of the car w = 30 000 N, and the angle of inclination $\theta = 0.1$ radians. If the maximum power of the car $P_{max} = 10$ kW, then find the maximum speed of the car. (Use the small-angle approximation $\sin\theta \approx \theta$.) Choose one of the following:

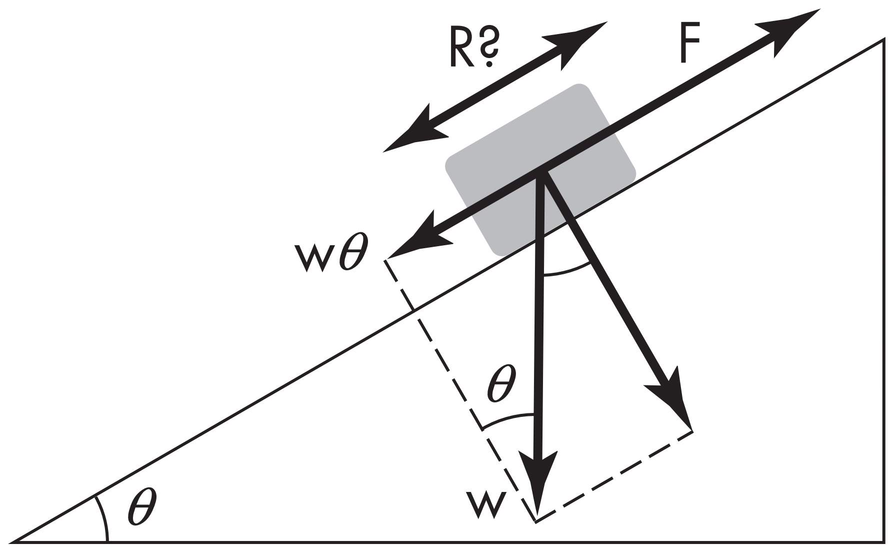
A	$v_{max} \approx P_{max}/(w\theta + R)$
B	$v_{max} \approx P_{max}/(w\theta - R)$
C	$v_{max} \approx P_{max}/(w\theta)$
D	$v_{max} \approx P_{max}/R$

Hint: $P = F \cdot v$.

Extra: Determine v_{max} in m/s.

Explanation: For the car moving uphill, **R** is pointing downhill. Because the car is moving with a constant speed, the net uphill force equals the downhill force: $F = w\theta + R$ or $v_{max} = P_{max}/F$. Answer = A.

Explanation—extra:
$v_{max} \approx P_{max}/(w\theta + R) = 10\,000/(30\,000 \times 0.1 + 1000) = 2.5$ m/s.



We begin with a similar setup as in Question 8-7. A car is climbing up a hill with a constant speed v_0. The air resistance (the drag force) is R_0. On a flat road, if the air resistance does not change with speed, the maximum speed of the car will be given by $v_{max} = P_{max}/R_0$ when the maximum power of the engine is assumed to be the same for both cases. Now assume the air resistance is proportional to the speed: $R = (v/v_0)\,R_0$. Again P_{max} is fixed. The corresponding maximum speed on a flat road v'_{max} is given by which of the following?

A	B	C	D
$(v_{max} + v_0)/2$	$\sqrt{v_{max}v_0}$	$\sqrt{2v_{max}v_0}$	v_{max}

Hint: $P = Fv$.

Explanation: Taking into account the velocity-dependent drag force assumed, the flat road maximum speed $v'_{max} = P_{max}/R' = P_{max}/[R_0 v'_{max}/v_0] = v_{max}v_0/v'_{max}$. Solving for v'_{max} gives $\sqrt{v_{max}v_0}$. In other words, the actual maximum speed is the geometric mean between v_0 and v_{max}. Answer = B.