chapter 9 Gravitation

Circular orbits (Section 9.3)

1, 2, and 3 are simple exercises to supplement the quantitative calculations of Examples 4, 5, and 6 in Section 9.3.

1. Satellite near Earth's surface
2. Gravitational acceleration in outer space
3. Communication satellite above Planet X

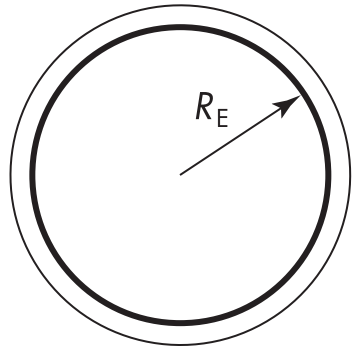
Elliptical orbits; Kepler's laws (Section 9.4)

4. Angular momentum of 2 orbits
   - Context in the text: After the discussion of Kepler's three laws in Section 9.4.

Energy in orbital motion (Section 9.5)

The following three problems are to supplement the examples given in Section 9.5:

5. Knock a planet out of the solar system—I
6. Knock a planet out of the solar system—II
7. Maximum height of a rocket



Determine the period of a satellite near the surface of the Earth ($r \approx R_E$):

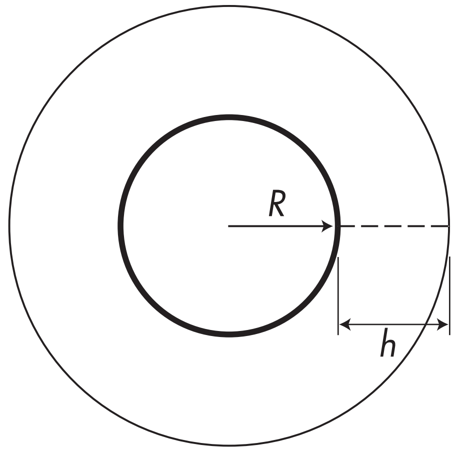
	T		
A	$2\pi\sqrt{\dfrac{g}{R_E}}$	C	$2\pi\sqrt{\dfrac{R_E}{g}}$
B	$\sqrt{\dfrac{g}{R_E}}$	D	$\sqrt{\dfrac{R_E}{g}}$

Hint: $g = v^2/R_E = \omega^2 R_E$, where $\omega = v/R_E$. This is the angular speed introduced in Question 10 for Chapter 6, with $\omega = 2\pi/T$.

Extra: Use $R_E \approx 6400$ km and $g = 10$ m/s^2 to estimate the period T. Should it be 1.5 seconds, 1.5 minutes, or 1.5 hours?

Explanation: Based on the hint, $v = \sqrt{gR_E}$. From this expression and $v = 2\pi R_E/T$, we find that $T = 2\pi\sqrt{\dfrac{R_E}{g}}$. Answer = C.

Explanation—extra: $T = 2\pi\sqrt{6400 \times 1000/10} \approx 6 \times 800$ s = 80 min. It is slightly over an hour.



Derive an expression for the gravitational acceleration at height $h = R$ above the Earth's surface. Express your answer in terms of g, the gravitational acceleration at the surface of the Earth. Notations: R and M represent the Earth's radius and mass. Choose one of the following:

	$g(r = 2R)$
1	g
2	$g/2$
3	$g/4$

Hint: $g(r) = GM/r^2$.

Explanation: $g = GM/R^2$, $g(r = 2R) = GM/(2R)^2 = g/4$.

Consider a communication satellite above the Earth. Let the satellite's period be $T = 1$ day and its orbital radius r_1. Imagine another communication satellite (called satellite X) above Planet X. For this problem assume that satellite X has the same period, $T_x = 1$ Earth-day, as the Earth satellite; further assume that the mass of Planet X is $M_x = 8\ M_E$. The orbital radius of satellite X should be given by which of the following?

A	B	C	D
$8r_1$	$6r_1$	$4r_1$	$2r_1$

Explanation: From Eq. (9.15) in the text, the satellite above the Earth satisfies the relation $T^2 = \dfrac{4\pi^2}{GM_E}\ r_1^3$ (1).

For Planet X, correspondingly, $T_X{}^2 = T^2 = \dfrac{4\pi^2}{GM_X}\ r_X^3$ (2). Equating the right sides of Eqs. (1) and (2), after

some cancellation, leads to

$r_1^3 = \dfrac{r_X^3\ M_E}{M_X} = \dfrac{r_X^3}{8}$, or $r_x = 2r_1$. Answer = D.

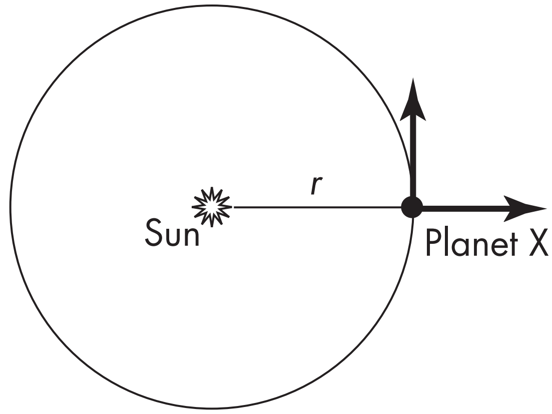
Consider two satellites orbiting the Earth. Orbit 1 is circular. Its orbital radius is a and its area is $A_1 = \pi a^2$. Orbit 2 is elliptical. The semimajor and semiminor axes are a and b respectively, and its area is $A_2 = \pi ab$. The two orbits have periods T_1 and T_2 and orbital angular momenta L_1 and L_2 respectively. If $T_1 = T_2$, what is the correct relationship between the satellites' angular momenta? (To work this problem we need to know the following: $L = rmv = 2m dA/dt = 2mA/T$. The first step is the definition of angular momentum. The second step uses the relation that area swept in time, dt, is $dA = rv dt/2$. See the discussion in the paragraph before Eq. (9.18). The last step uses Kepler's Second Law: dA/dt = constant.)

A	B	C
$L_1 > L_2$	$L_1 = L_2$	$L_1 < L_2$

Hint: Use the relation $L = 2mA/T$.

Explanation: Taking into account the givens ($T_1 = T_2$; the area of the circle is πa^2; that of the ellipse is πab), the ratio of angular momenta can be written like this:

$$\frac{L_1}{L_2} = \frac{2mA_1/T_1}{2mA_2/T_2} = \frac{A_1}{A_2} = \frac{\pi a^2}{\pi ab} > 1.\ \text{Answer} = \text{A.}$$

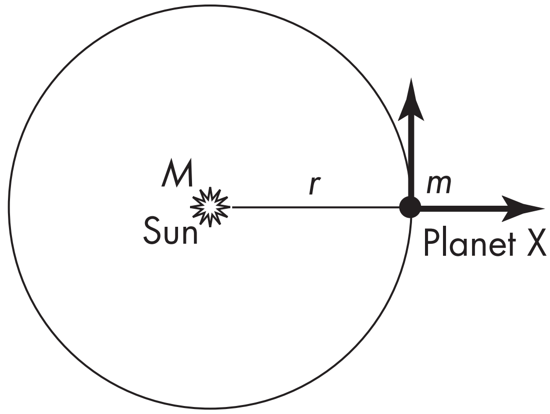


Consider a collision between a heavenly body and a hypothetical Planet X orbiting the Sun. (See the sketch.) Determine the minimum transfer of kinetic energy required to knock Planet X out of the solar system. (Notice that this kinetic energy transfer is not tied to a particular direction. For instance, among many choices, as indicated in the sketch, it could be along the tangential direction or along the radial direction.) Here we denote the mass of Planet X by *m* and the mass of the Sun by *M*. Which of the following descriptions is true?

	Minimum energy required
A	GmM/r
B	$GmM/(2r)$
C	GmM/r^2

Hint: Potential energy $U(r) = -GmM/r$.
Centripetal force $mv^2/r = GmM/r^2$.

Explanation: Denote the minimum kinetic energy transferred as ΔK. The condition for leaving the solar system is $U(r) + K + \Delta K = 0$.
Or $\Delta K = GmM/r - GmM/(2r) = GmM/(2r)$. Answer = B.



Begin with the same setup as the previous problem: A hypothetical Planet X is orbiting the Sun. Now consider the collision between a heavenly body and Planet X. Determine the ratio v_f/v_i where v_f is the final speed of Planet X (defined with respect to the Sun) immediately after the collision, and v_i is the orbital speed before the collision. Again the mass of Planet X will be denoted by m and that of the Sun by M. Choose one of the following solutions:

A	B	C	D
$\sqrt{2}$	2	$\sqrt{3}$	3

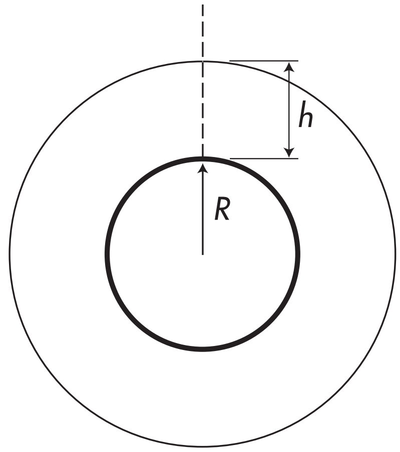
Hint: Potential energy $U(r) = -G\, mM/r$.
Centripetal force $mv^2/r = GmM/r^2$.

Explanation: Analogous to Eq. (9.23):

During orbital motion: $K_i = \dfrac{mv_i^2}{2} = \dfrac{GmM}{2r}$ (1)

Immediately after collision: $K_f = \dfrac{mv_f^2}{2} = \dfrac{GmM}{r}$ (2)

The ratio of Eq. (2) to Eq. (1) leads to: $v_f/v_i = \sqrt{2}$. Answer = A.



Consider launching a rocket from the surface of the Earth. It quickly reaches its initial kinetic energy K_i near the surface of the Earth. If it finally arrives at a maximum height above the surface $h = R/2$, determine the initial kinetic energy. Denote the mass of the Earth by M, the radius of the Earth by R, and the mass of the rocket by m. Choose one of the following:

A	B	C	D
mgh	(3/4) *mgh*	(2/3) *mgh*	(1/2) *mgh*

Hint: The potential energy at Earth's surface is $-GmM/R$. At the maximum height it is $-GmM/(h + R)$.

Explanation: Conservation of energy implies that $K_i + U(R) = K_f + U(h + R) = U(h + R)$. In other words, the initial kinetic energy required is the difference between the final potential energy and the initial potential energy.

Thus $K_i = -\dfrac{GmM}{R + h} + \dfrac{GmM}{R} = \dfrac{GmMh}{R_E(R + h)} = \dfrac{2}{3}\,mgh$. In the last step $g = \dfrac{GM}{R^2}$ and $h = R/2$ were used. Answer = C.