chapter 10 Systems of Particles

Momentum (Section 10.1)

1. Two blocks and a spring
2. Impulse delivered by an incline
3. Ball hits the wall
4. Two-dimensional inelastic collision

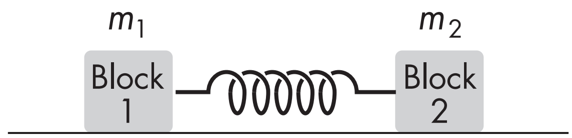
Center of mass (Section 10.2)

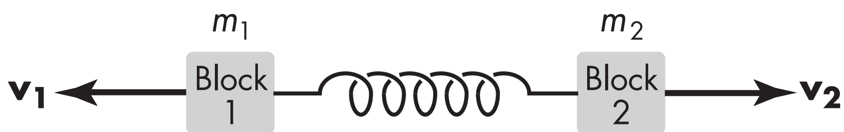
5. A "combination rule"
6. A boy in a boat

Energy and momentum in a two-body system (Section 10.4)

7. Block sliding down a curve
8. Daring professor

Contents





Consider two blocks held together at rest with a compressed spring between them as shown in the top sketch. Assuming that $m_2 = 2m_1$, find the ratio of their speeds v_2/v_1 once they are released. Choose one of the following answers:

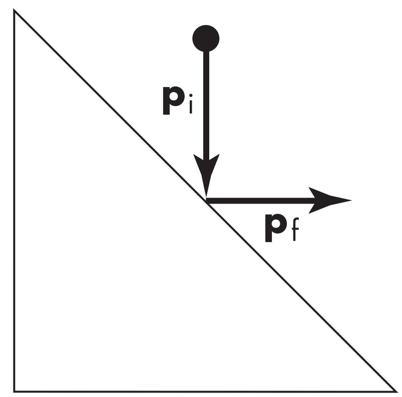
	A	B	C	D
v_2/v_1	1/2	1	2	4

Hint: There is no external force acting on the two-block system. The sum of final momenta equals 0 because initially the system is at rest:
$m_1\mathbf{v}_1 + m_2\mathbf{v}_2 = \mathbf{p}_1 + \mathbf{p}_2 = 0$.

Extra: Determine the ratio of the kinetic energies after the blocks are released: K_2/K_1. Kinetic energy can be calculated as follows: $K = mv^2/2 = p^2/(2m)$.

Explanation: $v_2/v_1 = m_1/m_2 = 1/2$. Answer = A.

Explanation—extra: $K_2/K_1 = [p_2{}^2/(2m_2)][p_1{}^2/(2m_1)] = m_1/m_2 = 1/2$.

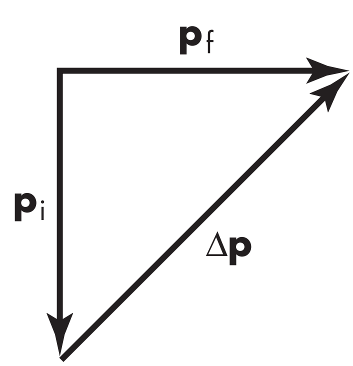


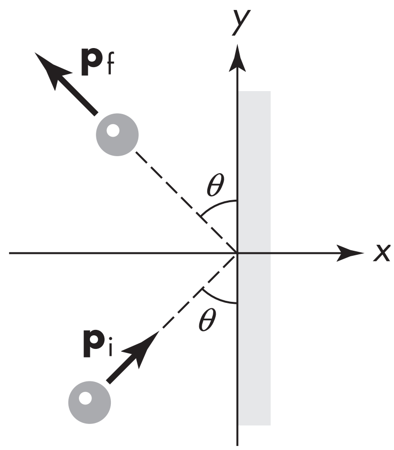
Consider the deflection of a ball by a 45° incline. The ball bounces off horizontally, and we assume it does not lose any kinetic energy during the deflection. Determine the impulse vector delivered by the incline to the ball. The impulse vector imparted on the ball by the plane is defined by $\Delta\mathbf{p} = p_f - p_i$. Choose one of the following:

	A	B	C	D
Direction	↗	↙	↗	↙
Δp	p_i	p_i	$\sqrt{2}\ p_i$	$\sqrt{2}\ p_i$

Hint: Sketch the vector diagram first. For the collision, $p_f = p_i$.

Explanation: From the sketch we see that Answer = C.

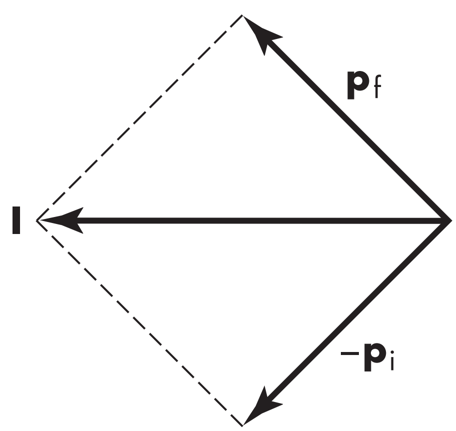


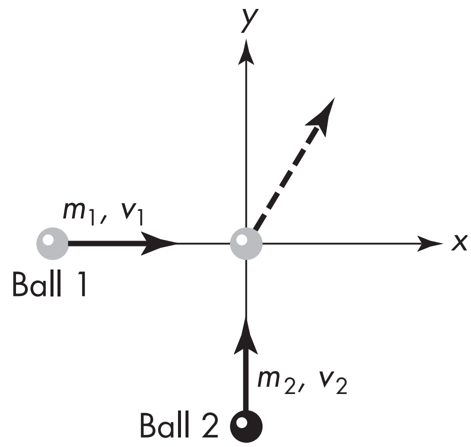


A steel ball strikes a wall with an initial momentum p_i at an angle θ with the surface. It bounces off with the momentum p_f at the same speed and angle. Sketch a vector diagram to determine the direction of the impulse vector $\mathbf{I} = p_f - p_i$. Choose one of the following:

A	B	C	D
Positive *y*	2nd quadrant	Negative *x*	3rd quadrant

Explanation: From the sketch we see that Answer = C.



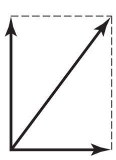


Consider the collision of two balls. Initially ball 1 is moving along the positive x direction with a speed $v_1 = 3v_0$. Ball 2 is moving along the positive y direction with speed $v_2 = 4v_0$. After the collision the balls are stuck together. If $m_1 = m_2 = m$, find the final speed v_f of the balls after the collision:

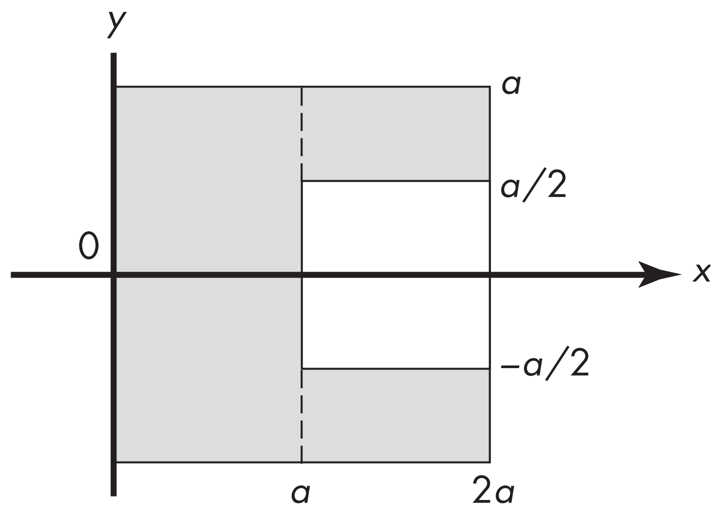
	A	B	C
v_f	$5v_0/2$	$5v_0$	$7v_0$

Hint: Because there is no external force, $\mathbf{p_f} = \mathbf{p}_{cm} = \mathbf{p}_1 + \mathbf{p}_2$.

Extra: Find K_f/K_i.

Explanation:  From the vector sum,
$\mathbf{p}_{cm} = \mathbf{p}_1 + \mathbf{p}_2$ gives $\mathbf{p}_{cm} = mv_0\sqrt{3^2 + 4^2} = 5mv_0 p_{cm} = mv_0\sqrt{(3^2 + 4^2)} = 5mv_0$. But $\mathbf{p_f} = (m_1 + m_2)v_f$, so $v_f = 5\mathbf{v_0}/2$.

Explanation—extra: $K_\mathbf{i} = K_1 + K_2 = m(3v_0)^2/2 + m(4v_0)^2/2 = 25\ mv_0{}^2/2$
$K_f = (2m)(5v_0/2)^2/2 = 25\ mv_0{}^2/4 = K_\mathbf{i}/2$.



Consider a letter "C" that is obtained by cutting a large square plate $2a \times 2a$ and removing a square $a \times a$ from the side, as shown in the sketch. Determine the x coordinate of the center of mass, x_{CM}:

A	B	C
$x_{CM} < a$	$x_{CM} = a$	$x_{CM} > a$

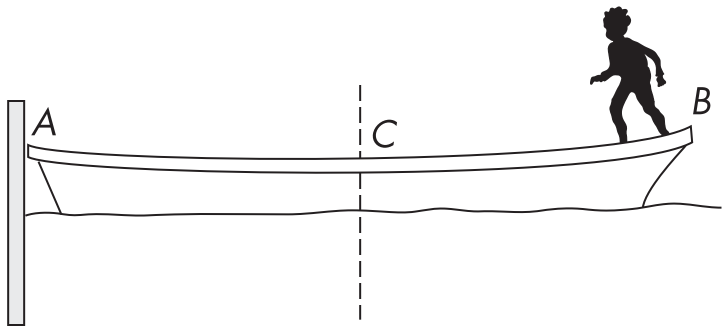
Extra: Calculate x_{CM}.

Hint for extra: Eq. (10.16) can be extended to include a hole. For the present problem, let mass #1 be the $2a \times 2a$ square and mass #2 the $a \times a$ hole. A hole contributes a negative term, which leads to the following "combination rule":

$x_{CM} = (m_1x_1 \pm m_2x_2)/(m_1 \pm m_2)$

Explanation: Intuitively, the location of the center of mass may be determined by balancing the plate with a fingertip. Due to symmetry, the x coordinate of the center of mass must be along the x axis. When there is a hole, the corresponding x_{CM} must be less than a, where a is the location of the center of mass when there is no hole.

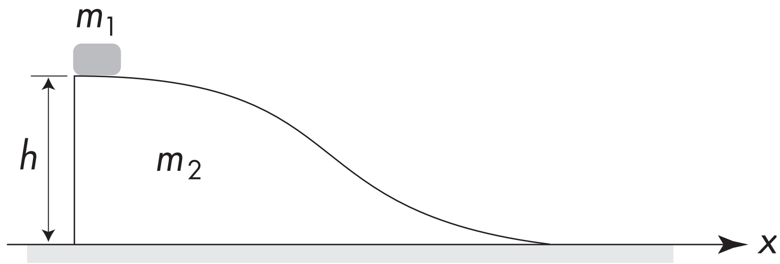
Explanation—extra: $x_{CM} = [4 \times a - 1 \times (3a/2)]/[4 - 1] = 5a/6$.



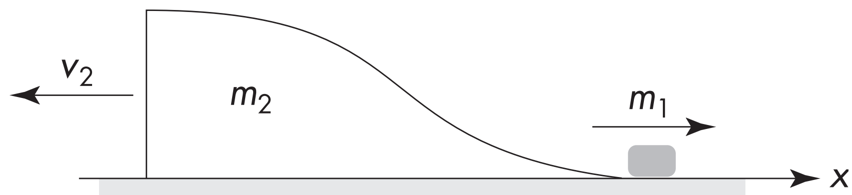
Consider a boat that has a length L and mass M. Assume it is initially touching the end of a pier and oriented perpendicularly to the shore, as shown. A, B, and C are points on the boat. A boy having a mass $m = M/3$ carefully walks from the far end B and to the middle of the boat C, which is also the center of mass of the boat. Assuming that the resistance between the boat and the water is negligible, the boat will now have moved from the edge of the pier. Determine the distance between the pier and the end of the boat A when the boy reaches the midpoint C:

A	B	C	D
$L/2$	$L/4$	$L/6$	$L/8$

Explanation: Because there is no external horizontal force acting on the combined system of the boat plus the boy, the location of the center of mass of this combined system must remain fixed with respect to the pier. At the beginning, the distance between the pier and the center of mass is $x_{CM} = [L \times M/3 + (L/2) \times M]/[M/3 + M] = 5\ L/8$. At the end, the center of mass of the combined system is at C on the boat, where $AC = L/2$. So the distance from the pier to point A is $x_{cm} - AC = 5L/8 - L/2 = L/8$. Answer = D.



(a)

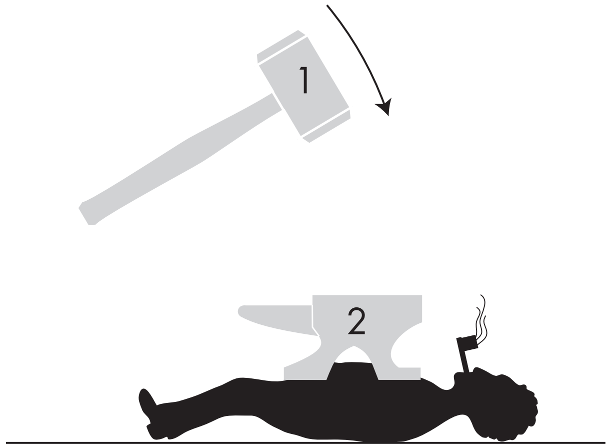


(b)

A block of mass m_1 is released from rest at the top of a curved, frictionless wedge of mass $m_2 = 3m$ as shown. The curved wedge sits on a frictionless horizontal surface. As the block slides down the curved surface, x_{cm}, the x coordinate of the center of mass of the combined system of block plus wedge, remains at rest. (Why?) What is the speed v_2 of the wedge with respect to the center of mass of the combined system after m_1 leaves the wedge?

A	B	C
$\sqrt{\frac{gh}{6}}$	$\sqrt{\frac{gh}{4}}$	$\sqrt{\frac{gh}{3}}$

Explanation: Because there is no external force along the horizontal direction and initially the center of mass of this system is at rest, the center of mass will remain at rest throughout the process. The vector sum of the block momentum $\mathbf{p}_1$ and the wedge momentum $\mathbf{p}_2$ in the center of mass must be zero: $\mathbf{p}_1 + \mathbf{p}_2 = 0$, or the magnitudes $p_1 = p_2 = p$. Conservation of energy implies that the potential energy released by the block equals the sum of the final kinetic energies: $m_1 gh = p^2/2m_1 + p^2/6m_1$. This leads to $p^2 = 3m_1^2 gh/2$. Because $p = m_2 v_2 = 3m_1 v_2$, then we have $v_2 = \sqrt{\frac{3m_1^2 gh}{2}}/3m_1 = \sqrt{\frac{gh}{6}}$.



In the physics demonstration illustrated here, a sledgehammer (1) hits an anvil (2) that is sitting on the daring professor lying down on the floor. Compare the momentum and kinetic energy of sledgehammer *before* the collision with those of the anvil *after* the collision.

A	$p_{1,i} \sim p_{2,f}$, $K_{1,i} \sim K_{2,f}$
B	$p_{1,i} \sim p_{2,f}$, $K_{1,i} >> K_{2,f}$
C	$p_{1,i} >> p_{2,f}$, $K_{1,i} \sim K_{2,f}$
D	$p_{1,i} >> p_{2,f}$, $K_{1,i} >> K_{2,f}$

Explanation: Answer = B.

- Conservation of momentum says $p_{1,i} + p_{2,i} = p_{1,f} + p_{2,f}$. For the present case, $p_{2,i} = 0$ and $p_{1,f} \approx 0$. So $p_{1,i} \approx p_{2,f}$. In other words, whatever momentum the sledgehammer carried was transferred to the anvil.
- The initial kinetic energy of the hammer is $K_{1,i} = \dfrac{p_{1,i}^2}{2m_1}$. The final kinetic energy of the anvil is $K_{2,f} = \dfrac{p_{2,f}^2}{2m_2}$. Because the numerators of the right sides for the two cases are comparable and the anvil is much more massive than the hammer, the kinetic energy of the anvil (2) is much smaller. The kinetic energy of the anvil is transferred to the professor as work. This work is expressed as the product of the average force F times the compression distance s: $W = Fs$. Because the energy transferred is relatively small, not much pain is expected.

 8. Daring Professor