chapter 11 Collisions

Impulse (Section 11.1)

1. Impulse delivered by the incline
   - Context in the textbook: This is an exercise of Eq. (11.4) in Section 11.1.

Elastic collisions in one dimension (Section 11.2)

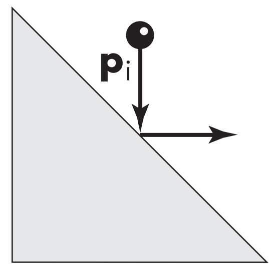
2. Elastic collision—I
3. Elastic collision—II
4. Sledgehammer and a ball
5. Collision with a pendulum

Inelastic collision in one dimension (Section 11.3)

6. Inelastic collision between a bullet and a ball

Collision in two dimensions (Section 11.4)

7. A perfectly inelastic collision in two dimensions

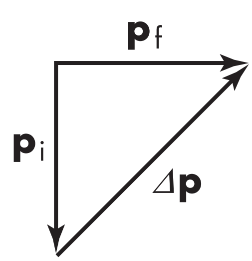


Consider the deflection of a falling ball by a 45° incline. The ball bounces off horizontally. Assume that this is an elastic collision. Determine the impulse delivered by the incline onto the ball:

	A	B	C	D
Direction	↗	↙	↗	↙
Δp	p_i	p_i	$\sqrt{2}\ p_i$	$\sqrt{2}\ p_i$

Hint: The impulse vector imparted on the ball by the plane is defined by $\Delta\mathbf{p} = \mathbf{p}_f - \mathbf{p}_i$. Sketch the vector diagram first.

Explanation: From the sketch, we see that Answer = C.



We have two balls with masses $m_1 = m_2 = m$. The ball m_1 with a speed $v_1 = v_0$ is approaching m_2, which is at rest ($v_2 = 0$). They undergo a head-on elastic collision. Denote the final velocities as v_1' and v_2'. Identify the correct pair of v_1' and v_2' values:

	A	B	C	D
v_1'	$v_0/2$	$v_0/3$	0	$-v_0/2$
v_2'	$v_0/2$	$2v_0/3$	v_0	$3v_0/2$

Hint: For an elastic collision, the following two conditions must be satisfied:

- Conservation of momentum: $\mathbf{p_i} = m\mathbf{v}_1 + m\mathbf{v}_2 = \mathbf{p_f} = m\mathbf{v}_1' + m\mathbf{v}_2'$.
- Conservation of kinetic energy: $K_f = K_i$.

Explanation: Notice that for all choices (in the x direction), $p_i = mv_1 + mv_2 = mv_0$ and $p_f = mv_1' + mv_2' = mv_0$. In other words, momentum is conserved for all four cases. Next check the condition $K_f = K_i$. The initial kinetic energy is given by $K_i = mv_0^2/2$. We need to work out K_f for all four choices. For instance, for the first choice A, $K_f = m(v_0/2)^2/2 + m(v_0/2)^2/2 = mv_0^2/4$. Do the same for the other three choices. All except C fail this condition. So Answer = C.

In Section 11.2, Eq. (11.13) and Eq. (11.14) are given for head-on elastic collisions between two balls of masses m_1 and m_2, where the ball with mass m_2 is initially at rest ($v_2 = 0$). These two equations can be written compactly as $v_i' = 2v_{cm} - v_i$ (where i = 1 or i = 2, depending on which ball's final velocity you are looking for). We will leave the proof as an exercise for the reader. Now envision m_1 approaching m_2, where $m_2 = 2m_1$ with a speed $v_1 = v_0$. After an elastic collision and using this new compact form, determine the final velocities v_1' and v_2':

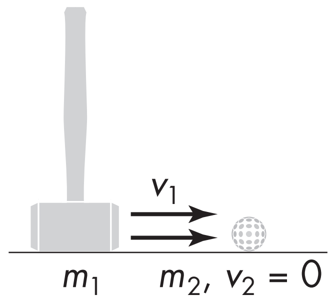
	A	B	C	D
v_1'	$-v_0/3$	$v_0/3$	$-v_0$	$-v_0/4$
v_2'	$2v_0/3$	$v_0/3$	$v_0/2$	$v_0/2$

Extra: Find the ratio of kinetic energies, K_f/K_i.

Explanation: The velocity of the center of mass is given by $v_{cm} = mv_1/(3m) = v_1/3 = v_0/3$. Based on the relation $v_i' = 2v_{cm} - v_i$, $v_1' = 2v_0/3 - v_0 = -v_0/3$, and $v_2' = 2v_0/3$. Answer = A. Among the incorrect choices, the reader should verify that

- Choice B corresponds to a perfectly inelastic collision.
- Both choices C and D violate the conservation of momentum.

Explanation—extra: Based on Answer A, $K_f = m(v_0/3)^2/2 + 2m(2v_0/3)^2/2 = (1/9 + 8/9)K_i$ or $K_f/K_i = 1$. This is expected because the present case corresponds to an elastic collision.



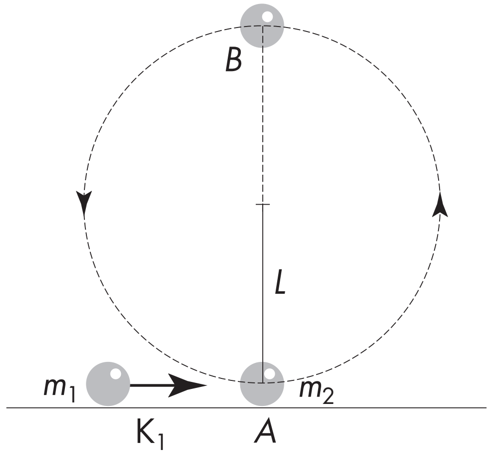
Consider an elastic, head-on collision involving a sledgehammer of mass $m_1 = 10$ kg hitting a golfball of mass $m_2 = 10$ g. Assume that $v_1 = 10$ m/s and $v_2 = 0$. Using the relations $m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2'$ and $v_i' = 2v_{cm} - v_i$ (see the previous question), find the final speed v_2' of the golfball:

A	$v_2' \approx v_1 = 10$ m/s
B	$v_2' \approx 2v_1 = 20$ m/s
C	$v_2' > 3v_1 = 30$ m/s

Hint: Because $m_1 >> m_2$, verify that $v_{cm} \approx v_1$.

Explanation: Using $v_i' = 2v_{cm} - v_i$ with $i = 2$ and $v_{cm} \approx v_1 = 10$ m/s (because $m_1 >> m_2$), we have $v_2' = 2v_{cm} \approx 2v_1 = 20$ m/s. Answer = B.

Digression: It is instructive to look at the process in the frame that is moving along with the sledgehammer. In this frame, the hammer is stationary. We can visualize that the initial velocity of the ball is $-v_1$, and the final velocity of the ball is $+2v_1 - v_1 = +v_1$. In other words, in this frame, the ball elastically bounces off the stationary hammer. Now consider the case in which the mass of the hammer is arbitrary. In the frame moving along with the center of mass (that is, in the cm frame), the initial velocity of the ball is $-v_{cm}$, and the final velocity of the ball in the cm frame is shifted from its lab frame value v_2' by an amount -0 cm, i.e. $v_2' - v_{cm} = (2v_{cm} - 0) - v_{cm} = v_{cm}$. In other words, the ball bounces off the center of mass with its velocity changing from $-v_{cm}$ to $+v_{cm}$—that is, it bounces elastically from the center of mass.



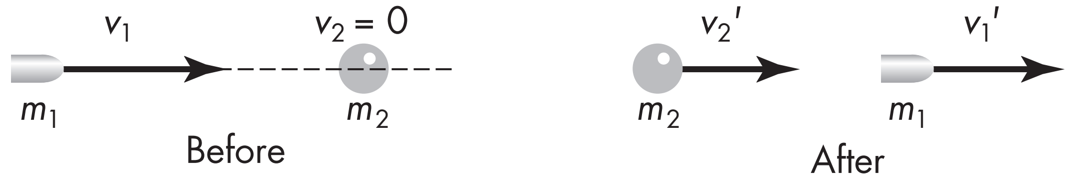
A ball of mass m_1 with initial kinetic energy K_1 is colliding with a pendulum bob of mass m_2 initially at rest. The pendulum bob is attached to a stiff rod of length L and is free to rotate around its other endpoint. Let $m_1 = m_2 = m$. Assume that the rod has negligible mass. Determine the minimum kinetic energy of the ball such that the bob can barely pass over the top at point B:

	A	B	C	D
Min. K_1	$3mgL/2$	$2mgL$	$5mgL/2$	$3mgL$

Hint: Immediately after a head-on elastic collision between two objects of equal mass where one is initially at rest, the initially moving object is stationary, and the initially stationary object moves off with kinetic energy equal to the kinetic energy of the originally moving object.
Extra: What is the minimum K_1 if the stiff rod is replaced by a string?

Explanation: In moving from A to B, energy is conserved—that is, the sum of kinetic energy plus potential energy remains constant, or $K_A + U_A = K_B + U_B$. With $K_B \approx 0$, $K_A = U_B - U_A = mg2L$. So the minimum value of $K_1 = K_A = 2mgL$. Answer = B.

Explanation—extra: At B, the resultant force responsible for the centripetal force is given by $mg + T = mv_B{}^2/L = 2K_B/L$, where T is the tension of the string. When the ball just barely passes through the point B, at that moment the tension of the string $T = 0$. It follows that $K_B = mgL/2$. So the minimum value of K_1 can be determined as follows:
$K_1 = K_A = U_B - U_A + K_B = mg2L + mgL/2 = 5mgL/2$.



A bullet with initial speed v_1 and mass m_1 is approaching a ball of mass m_2, initially at rest. Let $m_2 = 2m_1$. After piercing the ball, the bullet's final velocity is $v_1' = v_1/2$. Find v_2', the final velocity of the ball:

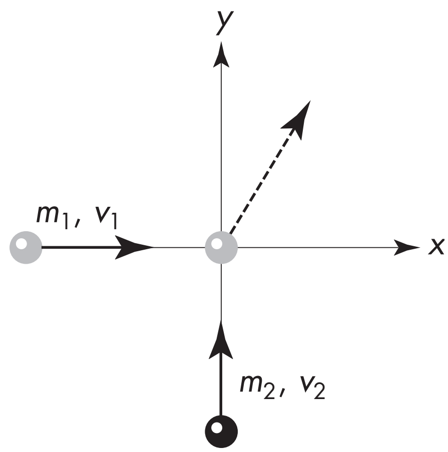
	A	B	C	D
v_2'	$v_1/8$	$v_1/4$	$v_1/3$	$v_1/2$

Hint: Use conservation of momentum: $m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2'$.

Extra: Find the ratio K_f/K_i.

Explanation: Using $m_2 = 2m_1$, $v_2 = 0$, and $v_1' = v/2$, the conservation of momentum equation $m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2'$ implies that $v_1 = v_1' + 2v_2' = v_1/2 + 2v_2'$. Solving for v_2' leads to $v_2' = v_1/4$. Answer = B.

Explanation—extra: The ratio of the kinetic energies may be determined as follows: $K_f = m(v_1/2)^2/2 + 2m(v_1/4)^2/2 = (1/4 + 1/8)[mv_1{}^2/2]$, where $K_i = \frac{mv^2{}_i}{2}$. So $K_f/K_i = 3/8$.



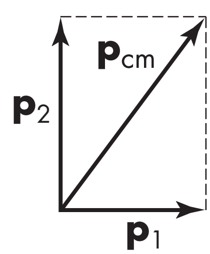
Consider the collision of two balls of masses m_1 and m_2, where $m_1 = m_1 = m$. Ball 1 is initially moving in the positive x direction with a speed $v_1 = 3v_0$. Ball 2 is initially moving in the positive y direction with a speed $v_2 = 4v_0$. After the collision, m_1 and m_2 are stuck together. What is the final speed of the combined system?

	A	B	C
v_f	$5v_0/2$	$5v_0$	$7v_0$

Hint: For a perfectly inelastic collision, $\mathbf{p}_f = \mathbf{p}_{cm} = \mathbf{p}_1 + \mathbf{p}_2$.

Extra: Find K_f/K_i.

Explanation: From the vector sum shown,



$\mathbf{p}_{cm} = \mathbf{p}_1 + \mathbf{p}_2$ gives $p_{cm} = mv_0\sqrt{(3^2 + 4^2)} = 5mv_0$, or $v_f = p_{cm}/(m_1 + m_2) = 5v_0/2$. Answer = A.

Explanation—extra: $K_i = K_1 + K_2 = m(3v_0)^2/2 + m(4v_0)^2/2 = 25mv_0{}^2/2$. $K_f = (2m)(5v_0/2)^2/2 = 25mv_0{}^2/4 = K_i/2$, or $K_f/K_i = 1/2$.