chapter 12 Rotation of a Rigid Body

Rotation about a fixed axis (Section 12.2)

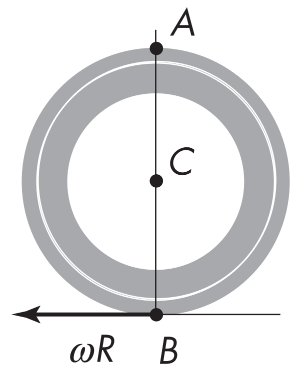
1. A rolling wheel—I
2. A rolling wheel—II

Rotation with constant acceleration (Section 12.3)

3. Coins on a lever

Moment of inertia and kinetic energy of rotation (Section 12.5)

4. Moment of inertia of four masses—I
5. Moment of inertia of four masses—II
6. Moment of inertia of a half-rod



Consider the rolling motion of a tire as shown in Fig. 12.10. In the reference frame of the automobile, the tire is rotating about its center with angular speed ω. Assume the wheel is rolling without slipping. With respect to the automobile, the ground is moving backward with a speed $v = \omega R$, where R is the radius of the tire. What is v_A, the speed of point A at the top of the tire, with respect to the ground? What is v_B, the relative speed between the contact point B (the lowest point of the tire) and the ground?

	A	B	C	D
v_A	$2v$	$2v$	v	v
v_B	v	0	v	0

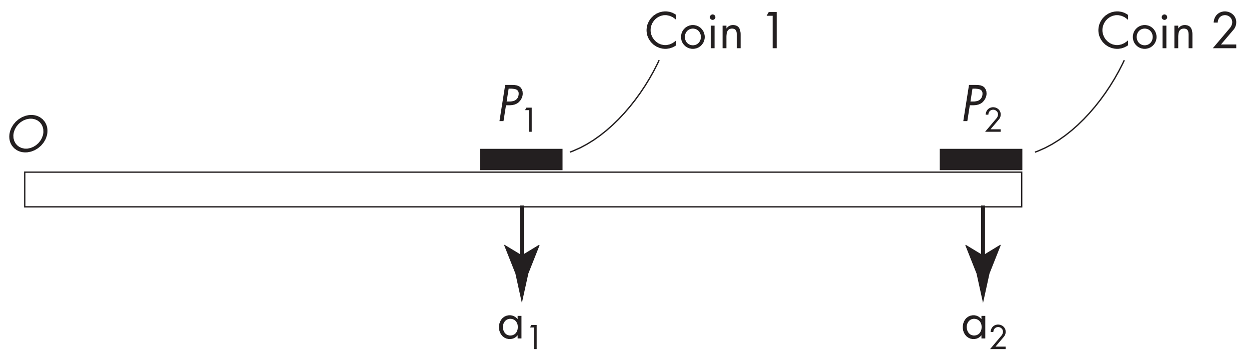
Explanation: The forward horizontal tangential velocity of A, the point at the top, defined with respect to the center, is $v_{AC} = v$. Adding v_{Cg}, the velocity of the forward motion of the center with respect to the ground, to v_{AC} gives the velocity of A defined with respect to the ground: $v_A = v_{Ag} = v_{AC} + v_{Cg} = v + v = 2v$. The velocity at point B, the lowest point of the tire, defined with respect to the center is given by $v_{BC} = -v$. The negative sign indicates that it is pointing backward. The velocity of B with respect to the ground is given by $v_B = v_{Bg} = v_{BC} + v_{Cg} = -v + v = 0$. Answer = B.

1. A Rolling Wheel—I

Consider the rolling motion of a tire as shown in the previous question, when the vehicle is moving with a constant speed v. Label the contact point on the tire at time $t = 0$ as C. Denote t_1 to be the next time when the point C contacts the ground, and d_1 to be the distance traveled during this interval. Assuming there is no slippage between the tire and the ground, determine t_1 and d_1:

	A	B	C	D
t_1	$2\pi R/v$	$2\pi R/v$	$\pi R/v$	$\pi R/v$
d_1	$2R$	$2\pi R$	$2R$	$2\pi R$

Explanation: The time t_1 is the period of rotation, which is $2\pi R/v$. Because the rolling motion is without slippage, during this time the distance the tire has rolled on the road must be the circumference, $2\pi R$. Answer = B.

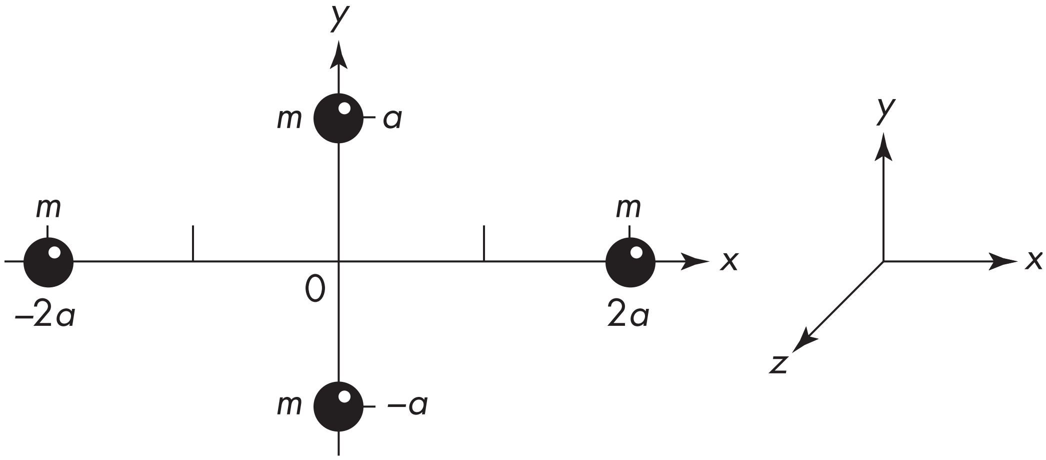


A uniform lever is pivoted at point O. The lever can rotate freely about O. Initially the lever is held in the horizontal position, with coin 1 resting at position P_1, where $OP_1 = L/2$, and coin 2 resting at position P_2, where $OP_2 = L$. Pivoting at O, the lever is rotating from the horizontal position at time $t = 0$ with an angular acceleration of $\alpha = 3g/2L$. Are the coins expected to stay on the stick *immediately* after the release of the stick? Here "s" denotes staying on and "d" denotes detachment:

	A	B	C	D
Coin 1	s	d	s	d
Coin 2	s	s	d	d

Hint: First determine the downward linear accelerations: a_1 at P_1 and a_2 at P_2. Then compare a_1 with g and a_2 with g.

Explanation: The downward acceleration of the lever at P_1 is $a_1 = \alpha(L/2) = 3g/4$. On the other hand, coin 2 can be accelerated by gravity with a downward acceleration of g. So coin 1 would fall faster than the lever. In other words, coin 1 will be *stuck* to the lever immediately after the release. At P_2, the downward acceleration of the lever is $a_2 = \alpha L = 3g/2$. The lever at P_2 is falling faster than coin 2, so coin 2 is expected to be detached from the stick right away. Answer = C.



The four identical masses shown are in the x–y plane, and the direction of the z axis is coming out of the paper. Find I_x:

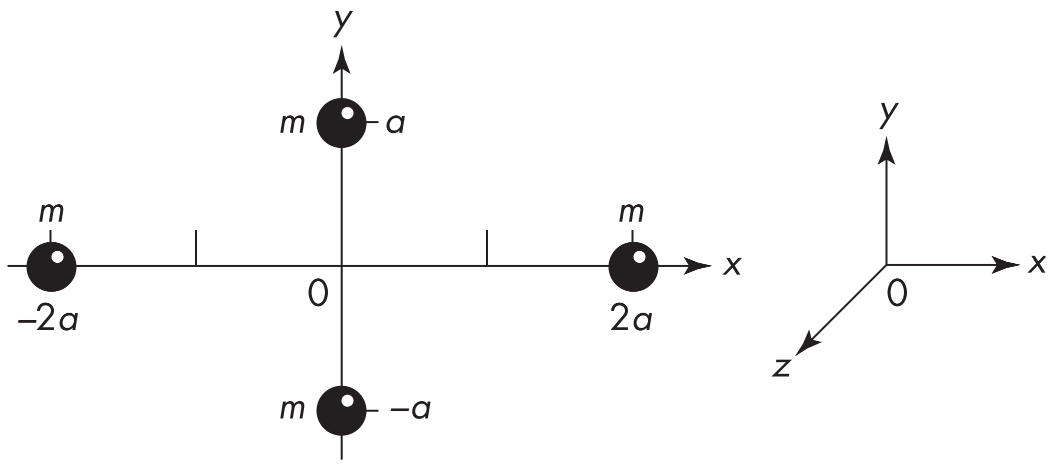
	1	2	3
I_x	ma^2	$2ma^2$	$8ma^2$

Hint: $I_x = \Sigma m_i x_i^2$, where x_i is the perpendicular distance between m_i and the rotating axis, which for the present case is the x axis. Similarly, $I_z = \Sigma m_i z_i^2$.

Extra: Determine I_x/I_z.

Explanation: $I_x = 2ma^2$.

Explanation—extra: $I_x/I_z = 2ma^2/[ma^2 + ma^2 + m(2a)^2 + m(2a)^2] = 1/5$.



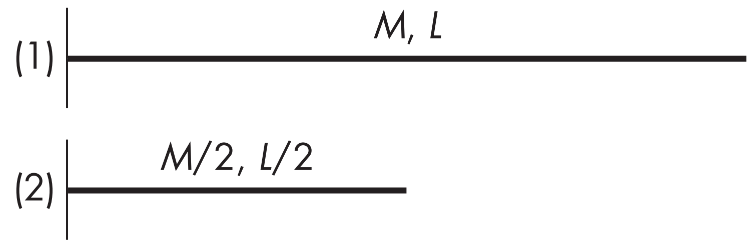
The four identical masses shown are in the x–y plane, and the direction of the z axis is coming out of the paper. Find K_y, the rotational kinetic energy where the rotational axis is chosen to be along the y direction and the angular speed is ω:

	A	B	C	D
K_y	$2ma^2\omega^2$	$4ma^2\omega^2$	$8ma^2\omega^2$	$16ma^2\omega^2$

Hint: $K_y = I_y\omega^2/2$.

Explanation: $I_y = 2 \times m \times (2a)^2$.

$K_y = I_y\omega^2/2 = 4ma^2\omega^2$. Answer = B.



In the two figures above, (1) represents a rod of length L and mass M, rotated around an endpoint; (2) represents half of the rod from (1), again rotated around its endpoint. In terms of L and M, find the moment of inertia I_2 of the half-rod:

	A	B	C
I_2	$ML^2/6$	$ML^2/12$	$ML^2/24$

Hint: From Table 12.3, the moment of inertia of a rod rotated about an endpoint is $I = \frac{1}{3} ML^2$.

Explanation: For the half-rod with its mass $m_2 = m/2$ and its length $L_2 = L/2$, $I_2 = \frac{1}{3} M_2 L_2^2 = \frac{1}{3}\left(\frac{M}{2}\right)\left(\frac{L}{2}\right)^2 = \frac{ML^2}{24}$. Answer = C.

An alternative derivation: Also from Table 12.3, the moment of inertia of the whole rod about a rotational axis that passes through its center of mass is $I_{cm} = ML^2/12$. By inspection, $I_{cm} = 2I_2$, or $I_2 = I_{cm}/2 = ML^2/24$.