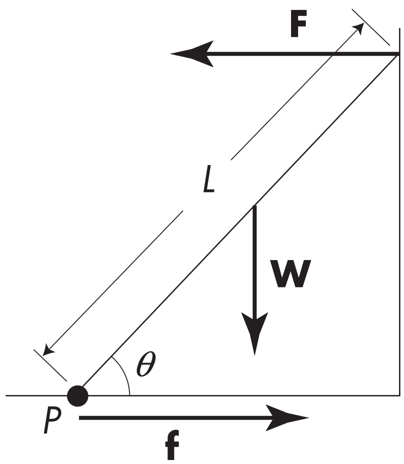
chapter 14

Statics and Elasticity

Static equilibrium (Section 14.2)

1. A leaning ladder
2. Solid sphere within a wedge
3. Holding a sphere
4. Two ladders
5. Two crane-and-weight systems
6. What will happen to the ladder?



A ladder of length L is leaning against a smooth wall. Friction between the ladder and the floor holds the ladder in place. Determine the counterclockwise torque due to $\boldsymbol{F}$, the normal force from the wall, and the counterclockwise torque due to $\boldsymbol{W}$, the weight of the ladder, about the pivot point P at the base of the ladder.

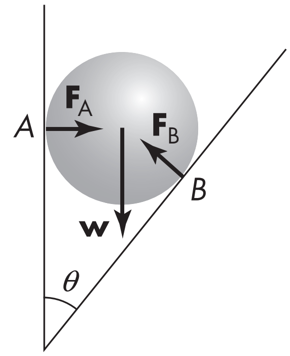
	Counterclockwise torque about P	Clockwise torque about P
A	$FL \sin\theta$	$W \times \frac{L}{2} \cos\theta$
B	$FL \cos\theta$	$W \times \frac{L}{2} \cos\theta$
C	$FL \sin\theta$	$W \times \frac{L}{2} \sin\theta$
D	$FL \cos\theta$	$W \times \frac{L}{2} \sin\theta$

Extra: For a ladder to be held in place, $F \le f_s^{max} = \mu W$, where f_s^{max} represents the maximum static friction between the ladder and the floor and μ is the coefficient of static friction. Show that this implies that $\mu \ge 1/(2 \tan\theta)$.

Explanation: By inspection, Answer = A.

Explanation—extra: Equilibrium of the frame in the horizontal direction implies that $F \le f_s^{max} = \mu W$. The torque equation is $(WL \cos\theta)/2 = FL \sin\theta \le \mu WL \sin\theta$. Solving for μ leads to $\mu \ge 1/(2 \tan\theta)$.

1. A Leaning Ladder

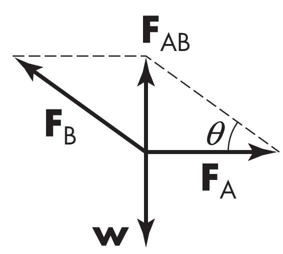


Consider a solid sphere of radius R and mass m placed in a wedge, where one wall is vertical and the other wall has an angle θ with respect to the vertical wall. The walls are smooth. Sketch a "free-body" diagram for the ball, which has $\mathbf{F}_A$ and $\mathbf{F}_B$ (the forces acting on the contact points A and B respectively) and weight $w = mg$. Then use the "free-body" diagram to choose the correct expression here:

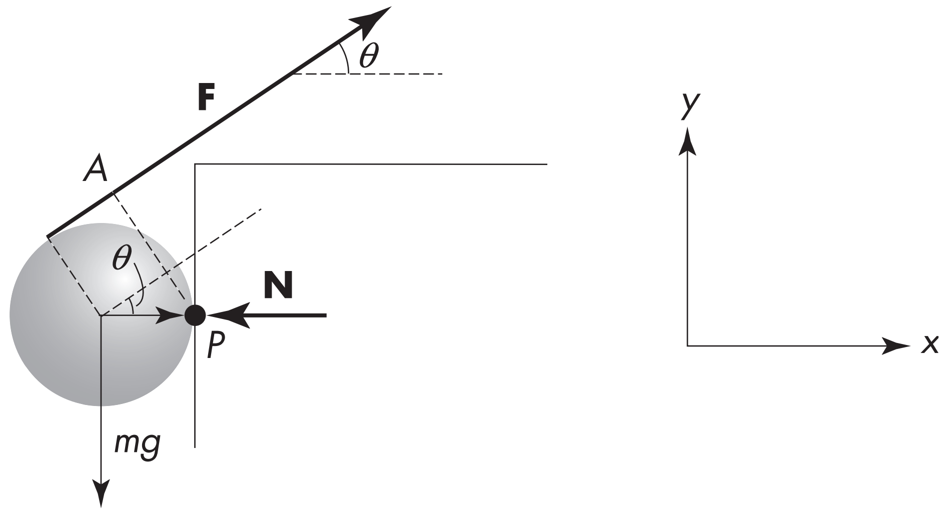
A	$mg = F_A \sin\theta$
B	$mg = F_A \cos\theta$
C	$mg = F_A \tan\theta$

Extra: Find a relationship among mg, F_A, and F_B.

Explanation: As shown in the sketch the upward resultant for a vector $\mathbf{F}_{AB}$ contributed by $\mathbf{F}_A$ and $\mathbf{F}_B$ cancels the downward weight w. From the sketch, $\tan\theta = mg/F_A$. So Answer = C.



Explanation—extra: By inspection, $F_B{}^2 = (mg)^2 + F_A{}^2$.



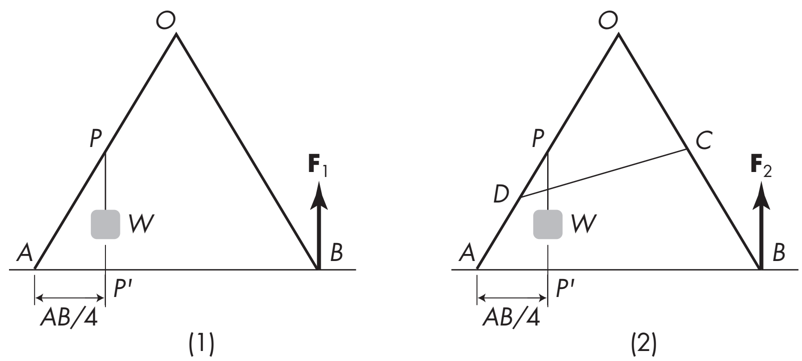
Consider a solid sphere of radius R and mass m held against a wall by a string pulled at an angle θ with the horizontal. Let F represent the tension of the string. Determine the torque equation for the sphere about the point P:

A	$Rmg = RF$
B	$Rmg = R(1 + \sin\theta)\ F$
C	$Rmg = R(1 + \cos\theta)\ F$

Extra: Find the force equation for the sphere in the x direction.

Explanation: About P, the clockwise torque is $\tau_{cw} = PA \times F$, where $AP = R(1 + \sin\theta)$ and the clockwise torque $\tau_{ccw} = Rmg$. So the equation is $\tau_{ccw} = \tau_{cw}$. Answer = B.

Explanation—extra: By inspection, the x direction equation is $N = F\cos\theta$, where N is the normal force pushing to the left.



Consider two ladders that are identical in all aspects except for one. The lengths of the sides are equal: $AO = BO$. A weight W is attached to the midpoint P ($AP = PO$). The masses of both ladders are negligible compared to W. They both rest on a frictionless horizontal floor. Their difference is as follows: For case (1) the junction at O is rigid—in other words, the angle AOB is fixed, say by welding. For case (2) the junction at O is loose. There is a connecting rope DC to secure the spread.

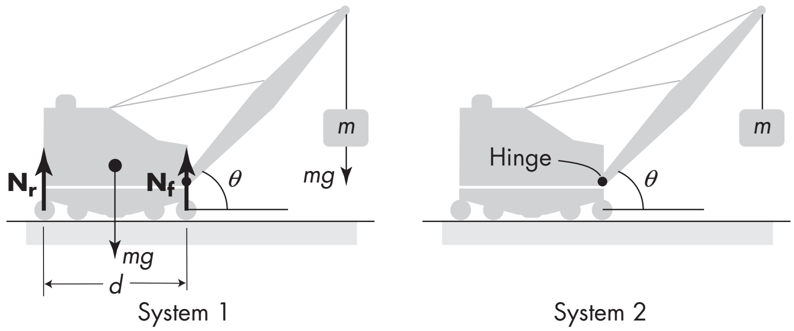
Compare F_1 and F_2—that is, the vertical forces at B for the two cases:

A	B	C
$F_1 > F_2$	$F_1 = F_2$	$F_1 < F_2$

Extra: Find F_2 in terms of W.

Explanation: For the setup of (1), the torque equation about the pivot point A is given by $AP' \times W = AB \times F_1$. For the setup of (2), there are additional forces because the string is pulling at C and at D. These two forces are equal and opposite, so they do not affect the torque equation. Therefore the torque equation for (2) is identical to that for (1). This leads to $F_2 = F_1$. Answer = B.

Explanation—extra: For case (2), the torque equation about the pivot point A is $AP' \times W = (AB/4) \times W = AB \times F_2$. This gives $F_2 = W/4$.



Two crane-and-weight systems are identical in all aspects except the following;

System 1: The crane arm is rigidly connected to the crane body.

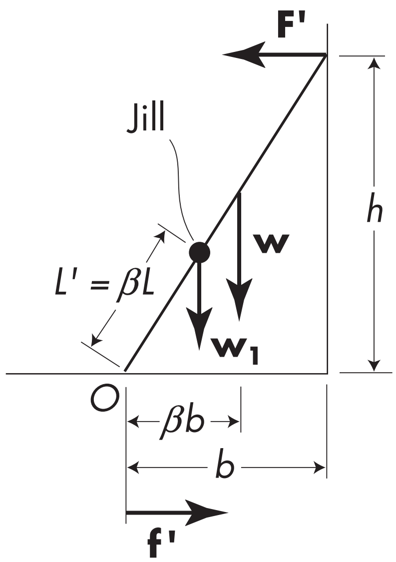
System 2: The connection is through a hinge together with two supporting wires with negligible masses.

Denote N_f and N_r to be the forces the ground exerts on the front and the rear wheels of system 1. The corresponding forces of system 2 are N_f' and N_r'.

Which of the following relation sets is true?

A	$N_f' = N_f,\ N_r' < N_r$
B	$N_f' = N_f,\ N_r' = N_r$
C	$N_f' > N_f,\ N_r' < N_r$
D	$N_f' > N_f,\ N_r' = N_r$

Explanation: The forces involved where the crane arm is rigidly attached to the crane body and where the supporting wires are attached to the crane arm and crane body are "internal forces." These internal forces always appear in pairs. Each pair has an action force and a reaction force. The two forces of each pair are collinear, and they always cancel each other out. For this reason only external forces will appear in the static equilibrium equations. Because the external forces for the present two cases are identical, the relationship among any respective forces for the two cases should also be identical. Answer = B.



A ladder of weight w is leaning against a smooth wall. Let F' refer to the normal force of the wall on the ladder. The coefficient of friction between the ladder and the floor is $\mu = 0.4$. The ladder is at its critical orientation when the height is h and the baseline is b. Jill of weight w_1 is standing at a point where $L' = \beta L$, with $\beta < 0.5$. What will happen to the ladder?

A	B	C
It becomes stable.	It remains at the critical point.	It will slip.

Explanation: About O, the torque equation is $hF' = \frac{b}{2} w + \beta b w_1$ (1). For the ladder to be stable requires that the horizontal force of friction from the floor $f' = F'$. Using Eq. (1) implies the following:

$$f' = \left(\frac{w}{2} + \beta w_1\right)\frac{b}{h} \leq f_{max}{}^{s} = \mu(w + w_1) \quad (2)$$

The first part of the given information says that for $w_1 = 0$, Eq. (2) becomes an equality:

$$\left(\frac{w}{2} + 0\right)\frac{b}{h} = \mu(w + 0), \text{ or } \mu = \frac{b}{2h} \quad (3)$$

Substituting Eq. (3) into the inequality (2) gives

$$\left(\frac{w}{2} + \beta w_1\right)\frac{b}{h} \leq \frac{b}{2h}(w + w_1), \text{ or } \beta \leq \frac{1}{2} \quad (4)$$

Eq. (4) implies that only when $\beta = 1/2$ does it correspond to the critical case. And as long as Jill is below the midpoint, the ladder-plus-Jill system is stable. Answer = A.