chapter 15 Oscillations

Simple harmonic motion (Section 15.1)

1. SHM: From initial x and v to A and ϕ
2. SHM: Projection of uniform circular motion
3. Mass–spring: The initial phase angle
4. Mass–spring: Projection of circular motion

Kinetic energy and potential energy in SHM (Section 15.3)

5. Total energy of a simple harmonic motion

Simple pendulum and physical pendulum (Section 15.4)

6. Physical pendulum
7. Simple harmonic oscillation of a loop
8. Torsional pendulum

Consider a simple harmonic motion (SHM) along the x axis centered about the origin. The displacement x and velocity v are given by the following (see Eq. (15.4) with δ replaced by ϕ):

$x = A\cos(\omega t + \phi)$ and $v = dx/dt = -A\omega \sin(\omega t + \phi)$.

At $t = 0$,

$x = x_0$ and $v = v_0$.

Determine the amplitude A:

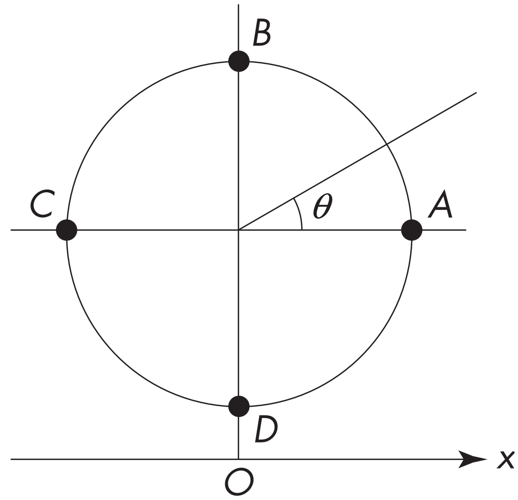
	A
A	x_0
B	$\frac{\omega}{v_0}$
C	$\sqrt{x_0^2 + \left(\frac{\omega}{v_0}\right)^2}$

Hint: At $t = 0$ there are two equations:
$x_0 = A\cos\phi$. (1)
$v_0 = -\omega A \sin\phi$. (2)

Extra: Express ϕ in terms of x_0 and v_0.

Explanation: This is the situation of two equations (1) and (2) with two unknowns: A and ϕ. Using $\cos^2\phi + \sin^2\phi = 1$, we may eliminate ϕ. More specifically, $(A\cos\phi)^2 + (A\sin\phi)^2 = x_0^2 + (v_0/\omega)^2 = A^2$. Solving for A leads to Answer = C.

Explanation—extra: Taking the ratio of Eq. (2) and (1) leads to the elimination of A: $\tan\phi = \sin\phi/\cos\phi = -(v_0/\omega)/x_0 = -v_0/(\omega x_0)$.

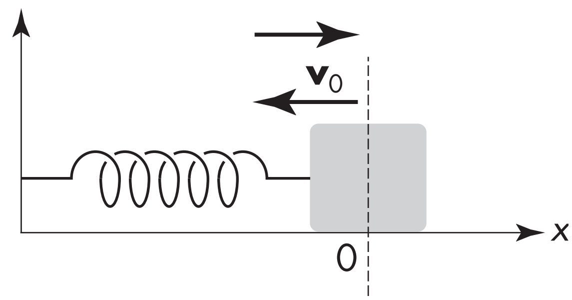


Consider a simple harmonic motion (SHM) $x = A \cos \theta$ as a projection of a uniform circular motion with $\theta = \omega t + \phi$. If at $t = 0$, $x = 0$ and $v = v_0 > 0$, determine ϕ:

	Point on the circle	ϕ
A	*A*	0
B	*B*	90°
C	*C*	180°
D	*D*	270°

Hint: Consider both of the initial conditions $x = 0$ and $v_0 > 0$.

Explanation: Because $x = 0$ when $t = 0$, we may choose either *B* or *D*. Notice that at *B* the velocity is along the negative *x* direction. But at *D* the velocity is along the positive *x* direction. So *D* is the correct choice—that is, at $t = 0$, $\theta = \phi = 270°$. Answer = D.



Consider a mass–spring system in which the mass oscillates according to simple harmonic motion: $x = A\cos(\omega t + \phi)$. At $t = 0$ the mass is at the equilibrium position moving to the left. Determine the phase angle ϕ:

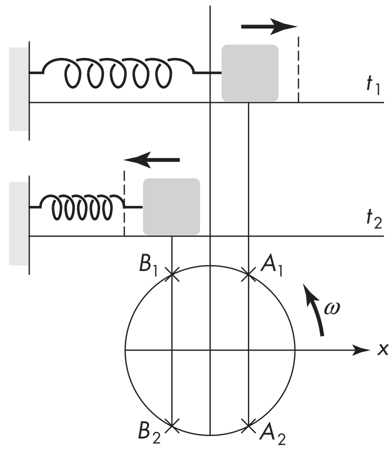
	ϕ
A	0
B	$\pi/2$
C	π
D	$3\pi/2$

Hint: $v = dx/dt = -\omega A\sin(\omega t + \phi)$.

Extra: Find an expression for the amplitude A of the periodic motion in terms of the maximum speed v_0 and the angular velocity ω.

Explanation: From the given information at $t = 0$, $x = 0 = A\cos\phi$, and $v = -|v_0| = -\omega A\sin\phi$. Because $\cos\phi = 0$, we have $\phi = \pi/2$ or $3\pi/2$. From the velocity equation, $|v_0| = \omega A\sin\phi$. This implies that $\sin\phi$ must be positive. In other words, $\phi = \pi/2$ is the correct choice. Answer = B.

Explanation—extra: The velocity equation gives $A = |v_0|/(\omega\sin\phi) = |v_0|/\omega$.



Consider the mass–spring system shown here at two different time t_1 and t_2, where the mass oscillates according to simple harmonic motion: $x = A\cos(\omega t + \phi)$. The motion is modeled as the projection of the circular motion shown at the bottom. Locate the point on the circle that corresponds to the time t_1:

	At t_1
A	A_1
B	A_2

Hint: Consider both the position and the velocity.

Extra: Which point on the circle corresponds to moment B?

Explanation: At the time t_1, as far the location of the mass is concerned, both A_1 and A_2 are possible. Because the velocity at moment A is positive, it limits the point A_2 to be the correct choice. Answer = B.

Explanation—extra: At the time t_2, the velocity of the mass is negative. By inspection, B_1 is the correct choice.

Consider a mass–spring system in which the oscillation is described by $x = A \cos \omega t$. The kinetic energy is $K = m(dx/dt)^2/2$. The potential energy is $U = kx^2/2$. The maxima are $K_{max} = m(\omega A)^2/2$ and $U_{max} = kA^2/2$. Which choice here gives the total energy of oscillation?

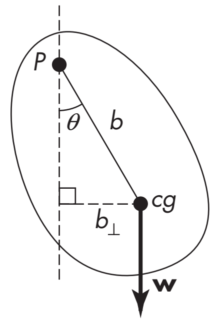
A	$E = K_{max} = U_{max} = m(\omega A)^2/2$
B	$E = K_{max} + U_{max} = m(\omega A)^2$
C	$E = K_{max} + U_{max} = kA^2$

Extra: At which point during the oscillation do you expect the mass–spring system to have the most energy?

- When the spring is relaxed (at $x = 0$).
- When the spring is fully stretched (at $|x| = x_{max}$).

Explanation: The total energy $E = K + U$ of the mass–spring system is a conserved quantity. E remains at the same value throughout the oscillations. When the mass passes the point $x = 0$, its potential energy is 0 and its kinetic energy is at its maximum. At the maximum stretch, its potential energy is at its maximum and its kinetic energy is 0. Answer = A.

Explanation—extra: Because E stays the same throughout the oscillations, the total energy of the mass–spring system at $x = 0$ is the same as the total energy at $|x| = x_{\mathbf{max}}$.



Consider a physical pendulum of mass m where P is the pivot point and b is the distance between P and the center of gravity. In a small θ approximation, $b_\perp = b \sin\theta \approx b\theta$, and the equation of motion is given by

$$\tau = I\alpha = Id^2\theta/dt^2 = -mgb\sin\theta \approx -(mgb)\theta \quad (1)$$

Determine the period T of this physical pendulum:

	A	B	C	D
T	$\sqrt{\dfrac{I}{\kappa}}$	$2\pi\sqrt{\dfrac{I}{mgb}}$	$\sqrt{\dfrac{\kappa}{I}}$	$2\pi\sqrt{\dfrac{mgb}{I}}$

Hint: For simple harmonic motion,

$$d^2s/dt^2 = -\omega^2 s \quad (2)$$

with $\omega = 2\pi/T$.

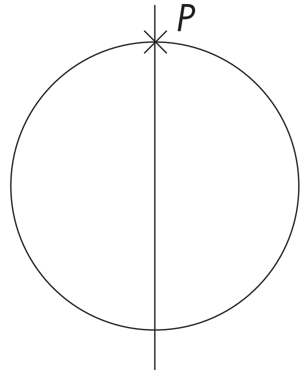
Extra: Find T of a simple pendulum that has a mass m and a length L.

Explanation: Comparing the equations of motion (1) and (2) with $\theta = s$ implies that $\omega^2 = mgb/I$. This leads to

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{mgb}}.$$ Answer = B.

Explanation—extra: For a simple pendulum, $I = mL^2$ and $b = L$. So

$$T = 2\pi\sqrt{\frac{L}{g}}.$$



The period of a physical pendulum is $T = 2\pi\sqrt{\frac{I}{mgb}}$, where m is the mass, I is the moment of inertia about the pivot point, and b is the distance between the pivot point and the center of gravity. Consider a circular loop oriented vertically where the pivot point P is at the top of the loop (see the sketch). Find b and I for a loop with radius r and mass m:

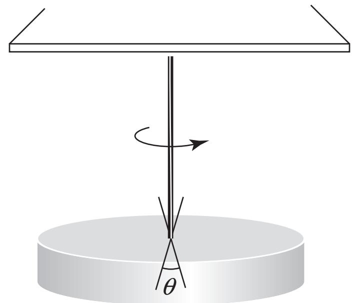
	A	B	C	D
b	r	r	$2r$	$2r$
I	mr^2	$2mr^2$	mr^2	$2mr^2$

Hint: Use the parallel axis theorem: $I = I_{\mathbf{cm}} + MD^{\mathbf{2}}$.

Extra: Find the period T of the oscillation of the loop.

Explanation: b is the distance between P and the center, so $b = r$. $I = I_{cm} + MD^2 = mr^2 + mr^2 = 2mr^2$. Answer = B.

Explanation—extra: $T = 2\pi\sqrt{\frac{I}{mgb}} = 2\pi\sqrt{\frac{2mr^2}{mgr}} = 2\pi\sqrt{\frac{2r}{g}}$.



A circular disk is suspended by a wire attached at the top of some fixed support. When the disk is twisted through some small angle θ, the twisted wire exerts a restoring torque on the body that satisfies $\tau = I\alpha = Id^2\theta/dt^2 = -\kappa\theta$, where κ is referred to as the *torsion constant* of the wire. Find the period of the oscillation:

	A	B	C	D
T	$\sqrt{\frac{I}{\kappa}}$	$2\pi\sqrt{\frac{I}{\kappa}}$	$\sqrt{\frac{\kappa}{I}}$	$2\pi\sqrt{\frac{\kappa}{I}}$

Hint: For simple harmonic motion, $d^2s/dt^2 = -\omega^2 s$, with $\omega = 2\pi/T$.

Explanation: The present equation of motion implies that $\omega^2 = \kappa/I$, and in turn, $2\pi\sqrt{\frac{I}{\kappa}}$. Answer = B.