chapter 16 Waves

Periodic waves (Section 16.2)

1. Traveling waves: shape and direction
2. Traveling velocity and oscillatory velocity
3. Traveling speeds along two wires

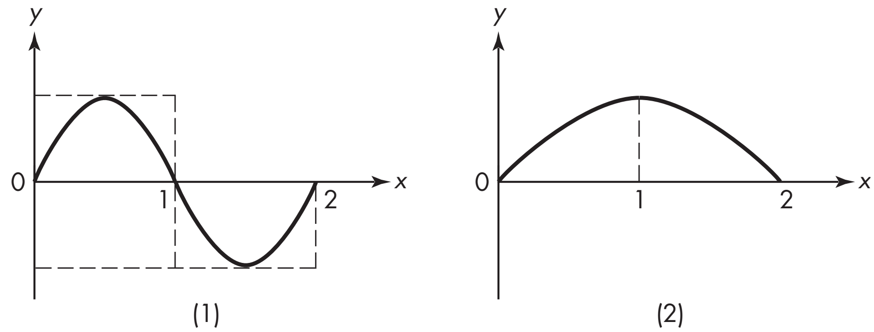
Superposition of waves (Section 16.3)

4. Beats

Standing waves (Section 16.4)

5. Super position of two opposing waves with same *k* and *w*
6. Standing waves: Locations of nodes and antinodes
7. Standing waves: Frequencies of harmonics

Consider a traveling wave: $y = 3\ \sin(\pi x + 3t)$. The two shapes (1) and (2) are indicated in the diagram here.



Choose the correct shape at $t = 0$, and traveling velocity:

	Shape at $t = 0$	Wave Velocity
A	(1)	~ 1
B	(1)	$\sim (-1)$
C	(2)	~ 1
D	(2)	$\sim (-1)$

Extra: What is the velocity amplitude of oscillations at any fixed value of *x*?

Explanation: At $t = 0$, $y = 3\ \sin(\pi x)$. As x varies from 0 to 2, the argument of the sine function varies from 0 to 2π. So (1) is the correct shape.

Alternative explanation for calculation of traveling velocity: We can obtain the traveling velocity by choosing y = constant and taking the time derivative of $y = 3\ \sin\ (\pi x + 3t)$. It gives $0 = 3\ \cos\ (\pi x + 3t)\ \frac{d}{dt}\ (\pi x + 3t)$, $\frac{d}{dt}\ (\pi x + 3t)$, or $\frac{dx}{dt} = \frac{-3}{\pi} \approx -1$. Answer = B.

Explanation—extra: $\frac{dy}{dt} = 9\ \cos(\pi x + 3t)$. So the velocity amplitude at y oscillations for any fixed x value is 9.

A traveling simple harmonic wave train is described by

$y = A \sin(kx + \omega t)$

where the wave number $k = 2\pi/\lambda$, and the angular frequency $\omega = 2\pi f$. The traveling wave velocity v_{wave} is given by which of the following?

	A	B	C	D
v_{wave}	λf	$-\lambda f$	λ/f	$-\lambda/f$

Hint: Traveling velocity may be determined in three steps:

1. Introduce a fixed-argument equation, which is given by "argument = constant."

2. Take the time derivative of this equation.

3. Solve for $dx/dt|_{argument}$, which is v_{travel}.

Extra: Show that the maximum velocity of the oscillations at any fixed x value is given by $\mathbf{v_{max}} = \omega A$.

Explanation: We follow the steps outlined in the hint.
From $d(kx + \omega t)/dt = 0$, we get $dx/dt = -\omega/k = -(2\pi/T)/(2\pi/\lambda)$
$= -\lambda/T = -\lambda f$. The wave is traveling in the negative x direction. Answer = B.

Explanation—extra: For fixed x, the oscillatory velocity is $v_{oscillations} = dy/dt = \omega A \cos(kx + \omega t) \equiv v_{max} \cos(kx + \omega t)$. So the amplitude of the velocity oscillations is given by $v_{max} = \omega A$.

Consider waves traveling along two different wires made from the same material. Use the following symbols:

Wire	Mass/Length	Tension	Diameter	Length	Speed
#1	μ_1	T_1	d_1	L_1	v_1
#2	μ_2	T_2	d_2	L_2	v_2

Given $d_2 = 2d_1$ and $L_2 = 2L_1$, determine the correct value for the ratio μ_1/μ_2:

	A	B	C	D
μ_1/μ_2	1/4	1/2	1	2

Hint: Mass/Length = (Density × Volume)/Length.

Extra: If $T_2 = 2T_1$, show that $v_2 = \dfrac{v_1}{\sqrt{2}}$. Use $v = \sqrt{\dfrac{F}{\mu}}$ [Eq. 16.14], where F is the tension and $\mu = M/L$.

Explanation: For a wire, Volume/Length = Cross section = $\pi(d/2)^2$. So for two identical materials with the same densities, the ratio is given by

$$\frac{\mu_1}{\mu_2} = \frac{\left[\pi\left(\dfrac{d_1}{2}\right)^2\right]}{\left[\pi\left(\dfrac{d_2}{2}\right)^2\right]} = \left(\frac{d_1}{d_2}\right)^2 = \frac{1}{4}.$$ Answer = A.

Explanation—extra: $v_2 = \sqrt{\dfrac{T_2}{\mu_2}} = \sqrt{\dfrac{2T_1}{4\mu_1}} = \dfrac{1}{\sqrt{2}} v_1.$

Two waves with equal amplitude but slightly different frequencies are traveling in the same direction. At a given point their displacements are described by $y_1 = A_0 \cos \omega_1 t$ and $y_2 = A_0 \cos \omega_2 t$. Evaluate $y = y_1 + y_2$. Identify the factor responsible for the beats:

A	$\sin[(\omega_1 - \omega_2)t/2]$
B	$\cos[(\omega_1 - \omega)t/2]$
C	$\sin[(\omega_1 + \omega_2)t/2]$
D	$\cos[(\omega_1 + \omega_2)t/2]$

Hint: $\cos \alpha + \cos \beta = 2 \cos \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2}$.

Extra: Show that the beat frequency is $f_{\text{beat}} = |f_1 - f_2|$.

Explanation: Using the trigonometric identity in the hint, $y = y_1 + y_2 = 2A_0 \cos\left(\frac{\omega_1 - \omega_2}{2} t\right) \cos\left(\frac{\omega_1 + \omega_2}{2} t\right)$. Because the difference between ω_1 and ω_2 is small, the first cosine factor corresponds to a low-frequency oscillation term. Its maxima give rise to beats. Answer = B.

Explanation—extra: Rewrite the first cosine term as $\cos(bt)$. Beats occur when $\cos(bt) = \pm 1$. Denote the time interval between two adjacent beats by T_{beat} (the beat frequency is f_{beat}). Then the corresponding angular interval is π—that is, $bT_{\text{beat}} = \pi$. So $b = \pi/T_{\text{beat}} = \pi f_{\text{beat}} = |\omega_1 - \omega_2|/2$. This implies that $f_{\text{beat}} = |f_1 - f_2|$.

Consider two traveling waves:

- $y_1 = A \sin(kx - \omega t)$
- $y_2 = A \sin(kx + \omega t)$

Determine the wave velocity of the superposed pattern:

	Wave Velocity of Superposed Pattern
A	Positive
B	Zero
C	Negative

Hint: Using $\sin \alpha + \sin \beta = 2 \sin\left(\frac{\alpha + \beta}{2}\right) \cos\left(\frac{\alpha - \beta}{2}\right)$, the superposed pattern is given by $y(x,t) = 2 \sin kx \cos \omega t$.

Extra: Determine the amplitude of oscillation at $x = 0$ and $x = \lambda/2$, with the wave number $k = 2\pi/\lambda$.

Explanation: From the superposed pattern $y(x,t)$ given in the hint, the first factor depends on x only and the second depends on t only. Here the x locations of the peaks and the valleys of the wave pattern do not vary with time. In other words, the wave pattern does not "travel" in t. So the traveling speed is zero. In other words, at a fixed x there is a fixed amplitude.
Answer = B.

Explanation—extra: We write $y = B \cos \omega t$. From the hint, the amplitude is a function of x—that is, $B = 2 \sin kx$. So at $x = 0$, $B = 0$. At $x = \lambda/4$, $kx = \pi/2$, or $B = 2 \sin \pi/2 = 2$.

Consider the superposition of two traveling waves:

- $y_1 = A_0 \sin(kx - \omega t)$
- $y_2 = A_0 \sin(kx + \omega t)$

Find the values of kx for which the amplitude vanishes:

	kx
A	$\pi/2, 3\pi/2, 5\pi/2, \ldots$
B	$\pi/2, 5\pi/2, 9\pi/2, \ldots$
C	$0, \pi, 2\pi, 3\pi, \ldots$
D	$0, 2\pi, 4\pi, \ldots$

Hint: $\sin\alpha + \sin\beta = 2\sin\dfrac{\alpha+\beta}{2}\cos\dfrac{\alpha-\beta}{2}$.

Extra: Determine the kx values at which the amplitude is $2A_0$.

Explanation: Using the identity given in the hint, $y = y_1 + y_2 = 2A_0 \sin kx \cos \omega t$. The zeros of the amplitude function: $2A_0 \sin kx$, occur at $kx = 0, \pi, 2\pi$, and so on. So Answer = C.

Explanation—extra: For the amplitude to be $2A_0$, where $\sin kx = \pm 1$, or $kx = \pi/2, 3\pi/2, 5\pi/2, \ldots$

Consider a string of length L fixed at both ends. Its fundamental frequency gives the middle C note with frequency $f_c = 264$ Hz.

Now consider a second string of the same length, again fixed at both ends. Its mass per unit length is half that of the first string, and its tension is twice as large.

The frequency f_2' of the second harmonic of the second string is given by which of the following?

	A	B	C	D
f_2'	f_c	$2f_c$	$3f_c$	$4f_c$

Hint: For normal modes of vibration,

- First harmonics ($n = 1$): The fundamental or lowest frequency is $f_1 = v/\lambda_1$.
- Overtones ($n \geq 2$): For the nth harmonics, $f_n = nf_1$.
- $v = \sqrt{\dfrac{F}{\mu}}$ for all harmonics.

Explanation: $f_2' = 2f_1'$. $f_1' = \dfrac{v'}{\lambda'} = \dfrac{1}{\lambda'}\sqrt{\dfrac{F'}{\mu'}}$. From the givens, both strings are fixed at both ends and have the same strength L. The fundamental mode corresponds to half of a wavelength, or $\lambda_1{}' = 2L$. Also from the givens, $F' = 2F$ and $\mu' = 0.5\mu$. This leads to $f_1' = \dfrac{1}{2L}\sqrt{\dfrac{2F}{0.5\mu}} = 2 \times \dfrac{1}{2L}\sqrt{\dfrac{F}{\mu}} =$ $2f_C$. $f_2' = 2f_1' = 4f_C$. Answer = D.