

***Pressure in static fluid (Section 18.4)***

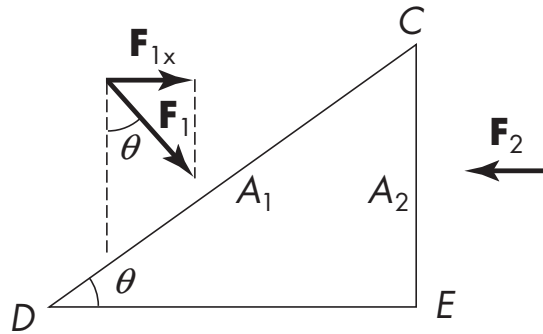
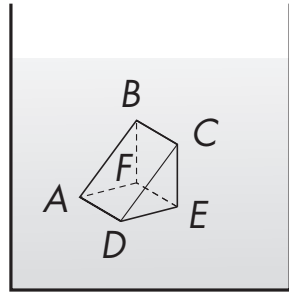
1. Pressure within a fluid body
2. Force on a dam
3. A hydraulic lift

***Archimedes' principle (Section 18.5)***

4. Archimedes' principle—I
5. Archimedes' principle—II
6. Helium balloon

***Bernoulli's equation (Section 18.6)***

7. Two water outlets
8. A siphon: steady flow at exit end
9. A fire extinguisher

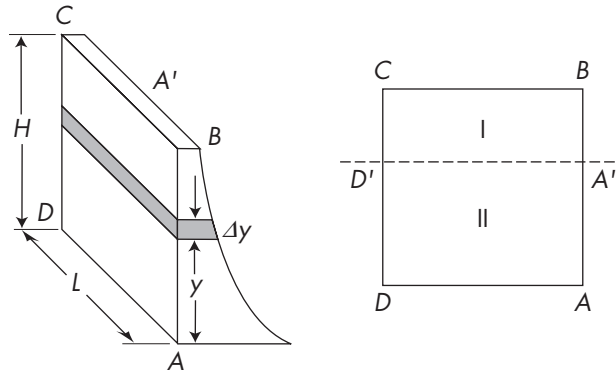


Consider a container filled with water. Imagine a small body of water in the shape of a rectangular prism. The inclined area  $ABCD$  is  $A_1$ . There is a normal force  $\mathbf{F}_1$  exerted on it. The vertical plane  $BCEF$  has an area  $A_2$ . There is a normal force  $\mathbf{F}_2$  exerted on it. Find the relationship between  $P_1 = F_1/A_1$  and  $P_2 = F_2/A_2$ .

A	B	C
$P_1 < P_2$	$P_1 = P_2$	$P_1 > P_2$

**Hint:** Equilibrium implies  $F_{1x} = F_2$ . Geometry:  $\sin \theta = A_2/A_1 = F_{1x}/F_1$  where  $\sin \theta = CE/CD = (CE \times BC)/(CD \times BC) = A_2/A_1$  was used.

**Explanation:** Equilibrium condition implies that  $F_{1x}/F_1 = F_2/F_1$ . So  $A_2/A_1 = F_2/F_1$ , or  $P_1 = P_2$ , consistent with Pascal's Law. Answer = B.



A dam has a height  $H$  and width  $L$ . The total force on the dam is given by which of the following?

	A	B	C
$F$	$\rho g H(HL)$	$(1/2)\rho g H(HL)$	$(1/4)\rho g H(HL)$

**Hint:**  $F = \int_0^H \rho g(H - y)L dy$

**Extra:** Divide the vertical area of the dam  $ABCD$  by the line  $A'D'$ , where  $AA' = DD' = (2/3)H$ . It gives two regions I and II (see the sketch). Show that the force in II is  $(4/9)\rho g H^2 L$ .

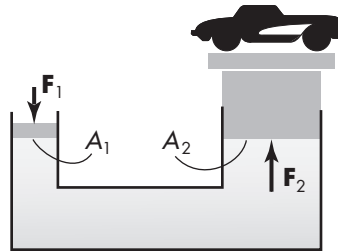
**Explanation:** From the integral defined in the hint, it gives Answer = B.

Intuitive reasoning: The total force equals the average pressure times the area, where the average pressure is given by the arithmetic mean of zero and the maximum value.

**Explanation—extra:** Using the arithmetic mean approach described in the explanation, the total force external in region I is given by

$$F_I = (1/2)\rho g(H/3) \times (LH/3) = (1/18)\rho g H^2 L. \text{ So the force in II is}$$

$$F_{II} = F - F_I = [(1/2) - (1/18)]\rho g H^2 L = (4/9)\rho g H^2 L.$$



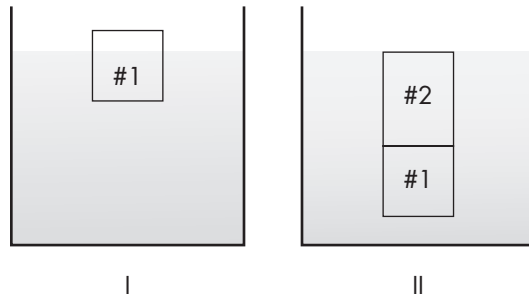
Consider the hydraulic lift shown in the sketch. A downward force  $\mathbf{F}_1$  is applied to the small piston, which results in an upward force  $\mathbf{F}_2$  at the large piston. The cross-sectional areas of the two pistons are  $A_1$  and  $A_2$ , respectively, where  $A_2/A_1 = 100$ . Compare the pressures  $P_1 = F_1/A_1$  and  $P_2 = F_2/A_2$ :

A	B	C
$P_1 < P_2$	$P_1 = P_2$	$P_1 > P_2$

**Extra:** Determine the ratio  $F_2/F_1$ .

**Explanation:** According to Pascal's Law,  $P_1 = P_2$ . Answer = B.

**Explanation—extra:**  $P_1 = F_1/A_1 = P_2 = F_2/A_2$ . So  $F_2/F_1 = A_2/A_1 = 100$ .



Consider two containers each with the same liquid of density  $\rho_L$  and blocks as shown.

- Case I: Block #1 is floating on the liquid. The submerged volume is  $xV_1 = V_L$ , where  $V_1$  is the volume of block #1 and  $V_L$  is the displaced volume of the liquid.
- Case II: Blocks #1 and #2 are submerged in the liquid.

Notations:

	Density	Volume
Block #1	$\rho_1$	$V_1$
Block #2	$\rho_2$	$V_2$

For Case I, choose the correct relationship between the two densities,  $\rho_1$  and  $\rho_L$ :

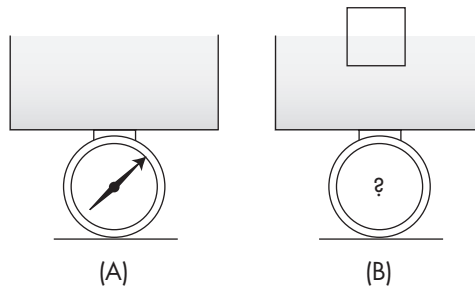
A	B
$\rho_1 = x\rho_L$	$x\rho_1 = \rho_L$

**Extra:** For case II, denote the densities and the volumes of blocks #1 and #2 to be  $\rho_1$  and  $\rho_2$  and  $V_1$ ,  $V_2$  respectively. Which one of the equations here is correct?

$$\rho_1 V_1 + \rho_2 V_2 = \rho_L (V_1 + V_2) \text{ or } \rho_1 V_2 + \rho_2 V_1 = \rho_L (V_1 + V_2)$$

**Explanation:** Because block #1 is floating, Archimedes' principle implies that the weight of the block equals the weight of the displaced liquid—that is,  $m_1 g = m_L g$ . This leads to  $\rho_1 V_1 = \rho_L V_L = \rho_L x V_1$ . Thus  $\rho_1 = x \rho_L$ . Answer = A.

**Explanation—extra:** Application of Archimedes' principle leads to the first choice here.



We have two identical containers filled with water to the brim. One has a piece of wood floating in it (case B), so its total weight is

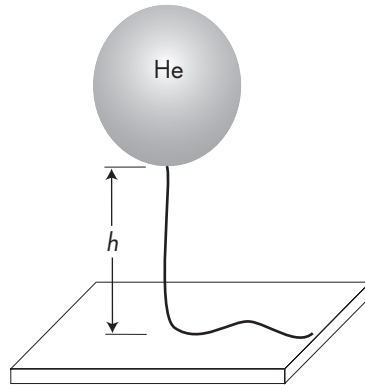
- A Greater than that of case A.
- B Less than that of case A.
- C The same as that of case A.

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**Explanation:** The difference in the two cases is that

- case A includes the weight of the “displaced water.”
- case B includes the weight of the block.

But according to Archimedes’ principle, these two weights are the same, so Answer = C.



Consider a balloon floating in the air (see the sketch). There is a string tied to the balloon. The string has a weight of  $W_{\text{string}}$  and a length  $L$ . Define the following set of symbols:

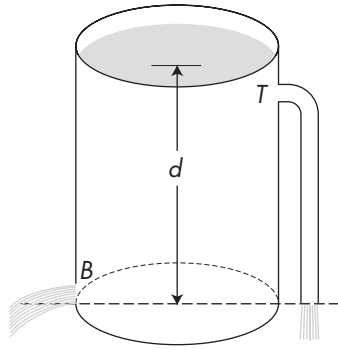
- $V_b$  = Volume of the inflated balloon.
- $W_b$  = Weight of the balloon (excluding the helium).
- $W_{\text{He}}$  = Weight of the helium in a volume  $V_b$ .
- $W_{\text{air}}$  = Weight of the air in a volume  $V_b$ .
- $h$  = Length of the string that is in the air.

Choose the correct relation (neglect the string volume):

Equation for the Buoyant Force	
A	$W_{\text{air}} = W_{\text{He}} + W_{\text{string}}(h/L)$
B	$W_{\text{He}} = W_b + W_{\text{string}}$
C	$W_{\text{air}} = W_b + W_{\text{He}} + W_{\text{string}}$
D	$W_{\text{air}} = W_b + W_{\text{He}} + W_{\text{string}}(h/L)$

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**Explanation:** Apply Archimedes' principle. The buoyant force equals  $W_{\text{air}}$ . This buoyant force lifts the balloon, the helium in the balloon, and the portion of the string that is in the air. Answer = D.

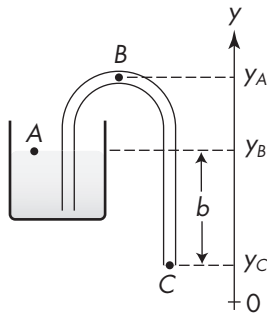


Consider a bucket of water with two outlets through which water is discharged. Water can be discharged from hole  $B$  some distance  $d$  below the water surface. It can also be discharged from the downspout connected to the vertical pipe. The vertical pipe begins at  $T$ , which is below the water surface. Assume the distance from the bottom of the downspout to the water surface is also  $d$ . If we neglect any friction effects, the water coming out of hole  $B$  has which of the following?

- A More speed than that coming out of the downspout
  - B Less speed than that coming out of the downspout
  - C The same speed as that coming out of the downspout.
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**Explanation:** Bernoulli's principle says that along a streamline tube, the "extended energy density"  $\varepsilon = P + \rho v^2 + \rho gh = \text{const}$ . For the present setup there are two distinct flows, or two streamline tubes. One flow involves some transport of the bulk of the water body which leads to the exit at the hole  $B$ . The other flow passes through  $T$  and exits at the downspout. Both flows share a common cross section at the top of the water surface. Bernoulli's principle implies the equality of the extended energy densities  $\varepsilon$  at the two exit ends. Both ends are at the same height and have the same external pressure. So their speeds are the same. Notice that  $T$  being below the water surface does not affect the conclusion here. Answer = C.





A siphon, which is a flexible tube with a circular cross section, is used to drain water from a tank as shown in the sketch. Assume the following:

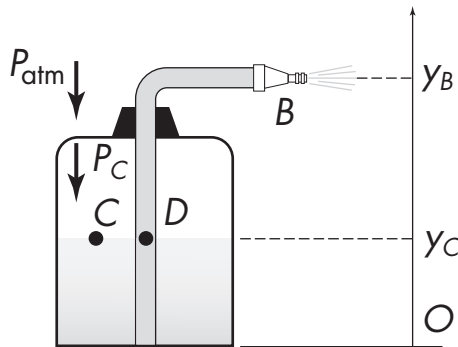
- A steady flow exists within the tube (from the water surface through the bend to the exit end).
- No friction affects the water.
- The water cannot sustain a negative pressure.

What is the maximum vertical distance between the top of the bend and the exit end, beyond which a continuous flow of water will not be possible?

	A	B	C
$h$	$\frac{P_{\text{atm}}}{\rho g} - b$	$\frac{P_{\text{atm}}}{\rho g}$	$\frac{P_{\text{atm}}}{\rho g} + b$

**Hint:** Use Bernoulli's principle:  $P + (1/2)\rho v^2 + \rho g y = \text{constant}$ .

**Explanation:** Applying Bernoulli's principle, gives  $P_C + \rho v_C^2/2 + \rho g y_C = P_B + \rho v_B^2/2 + \rho g y_B$ . Because  $v_C = v_B$ , we obtain  $\rho g (y_B - y_C) = P_C - P_B$ . The maximum drop without negative pressure occurs when  $P_B = 0$  in other words, when  $\rho g (y_B - y_C) = P_C = P_{\text{atm}}$ . So the maximum drop while maintaining a continuous flow is  $BC = P_{\text{atm}}/\rho g$ . Beyond this drop the water body within the BC section will no longer be continuous—that is, the occurrence of air bubbles may be expected. Answer = B.



Consider a fire extinguisher in which water is forced out of a pinhole nozzle by air pressure (see the figure). Denote the following:

- The pressure of the air in the bottle is  $P_C$ .
- The water density is  $\rho$ .
- The height of the nozzle from the water surface is  $h$ .
- $A_B$  is the cross section of the pinhole;  $A_D$  is the cross section of the tube.  $A_D \gg A_B$ .
- $v_D$  is the speed within the tube at the water surface, and  $v_B$  is the speed at the nozzle.

Find the relationship between  $v_D$  and  $v_B$ :

A	B	C
$v_D = v_B$	$v_D < v_B$	$v_D \ll v_B$

**Hint:** Use the continuity condition:  $v_D A_D = v_B A_B$ .

**Extra:** Show that  $P_C - P_{\text{atm}} = (1/2)\rho v_B^2 + \rho gh$ .

**Explanation:** Continuity implies that  $v_D = \frac{v_B A_B}{A_D}$ . Because  $A_D \gg A_B$ , this implies that  $v_D \ll v_B$ . Answer = C.

**Explanation—extra:** From Bernoulli's principle,  $P_C + \rho v_C^2/2 + \rho g y_C = P_B + \rho v_B^2/2 + \rho g y_B$ . Because  $v_C \ll v_D \ll v_B$ , the  $v_C$  term is negligible. In the present case,  $y_B - y_C = h$ , and  $P_B = P_{\text{atm}}$ . So Bernoulli's equation gives  $P_C - P_{\text{atm}} = (1/2)\rho v_B^2 + \rho gh$ .